Modal Models for Bradwardine's Truth

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Outline

INTRODUCTION

Thomas Bradwardine (≈ 1295–1349)

A page spread from Bradwardine's *Arithmetica Speculativa*

Bradwardine's proposal is that the Liar paradox, and indeed all such insolubles, are simply false, and not true. At the heart of his diagnosis lies his Thesis 2, that "If a proposition signifies itself not to be true or to be false, it signifies itself to be true and is false."[8] His proof depends on certain background definitions and postulates, which he sets out explicitly:

Definition 1 *A true proposition is an utterance signifying only as things are.*
Definition 2 *A false proposition is an utterance signifying other than things are.*

Postulate 1 *(Bivalence) Every proposition is true or false.*
Postulate 2 *Every proposition signifies or means contingently or necessarily everything which follows from it contingently or necessarily.*

Stephen Read, "The Liar Paradox from John Buridan back to Thomas Bradwardine"
Vivarium 40 (2002), 189–218. Excerpt from page 191

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will be used to *define* T.

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FACT: Everything is exactly one of *nondeclarative*, *true* and *false*.

Syntax:

Logic:

T *biconditionals*:

The concept T:

Syntax:	typed or type-free
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QUESTION: What is the conditional '$\rightarrow$' used in the axiom?

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– Nondeclarative sentences say *nothing*.
– True sentences say *everything that is the case*.
– False sentences say *everything*.

The classical axiom is not very discriminating.

The Modal Bradwardine Axiom

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The Modal Bradwardine Axiom

$$\Box(p \supset q) \supset \Box(x : p \supset x : q)$$

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However, everything declarative says every *necessity*, and anything saying an *impossibility* says everything.

Relevant Bradwardine Axioms

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It is vacuously true under the classical Bradwardine axiom (then λ says *everything*). I have counterexamples under *modal* and *relevant* axioms.

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But what do models of this theory look like?

THEORY

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Classical logic ($\wedge, \vee, \neg, \supset$) with propositional quantification ($\forall p$), and first-order quantification ($\forall x$), and an S5 modal operator ($\Box$).

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For simplicity, we will use Kripke models for constant domain quantified S5, in which accessibility is universal and the propositional quantifiers range over every set of worlds.

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This logic ($QS5^{\forall p}$) is not axiomatisable, but that's no problem, as we're looking for some neat models for a theory.

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But we want *interesting* models,

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in which we can express sentences like λ.

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Transparency rules out *reticent* models, but not *verbose* ones.

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(At least in the absence of context shifts, etc.)

Grounded Sentences

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This requirement makes modelling the theory more interesting.

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T-DEF	Tx is $(\exists p)(x : p) \wedge (\forall p)(x : p \supset p)$
TRANSPARENCY	$\ulcorner A \urcorner : A$
	It follows that $T\ulcorner A \urcorner \supset A$.
GROUNDED T-INTRO	$A \supset T\ulcorner A \urcorner$ for grounded A.

MODELS

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Our models treat ‘says that’ as a family of normal modal operators.

Simple Bradwardine Frames

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Given an interpretation of the atomic predicates (and an assignment of values to variables) this suffices to interpret the language of $QS5^{\forall p}$ in the usual way.

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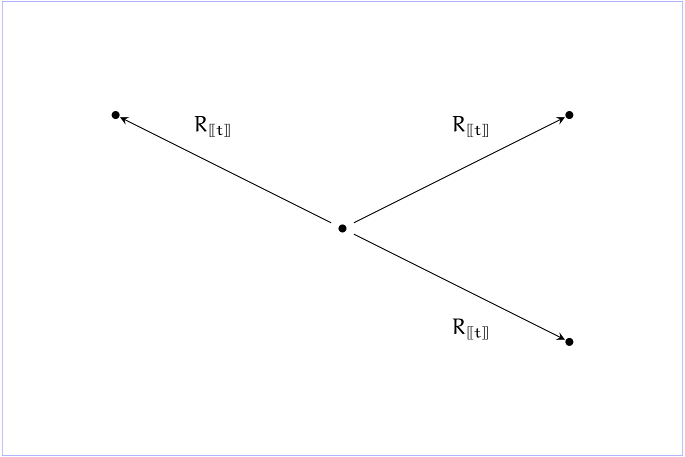
$\mathfrak{M}, \alpha, w \Vdash t : A$ iff $[\![t]\!]_\alpha \in D$ and $(\forall v \in W)(wR_{[\![t]\!]_\alpha}v \supset \mathfrak{M}, \alpha, v \Vdash A)$.


$R_{[[t]]}$
$R_{[[t]]}$
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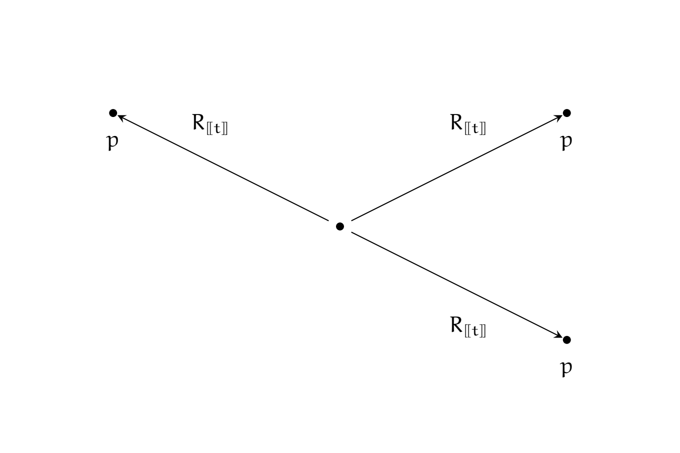
Example: non-reflexive points



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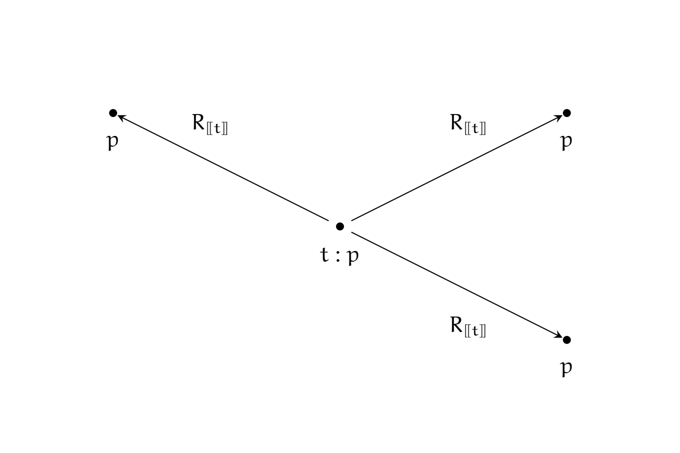

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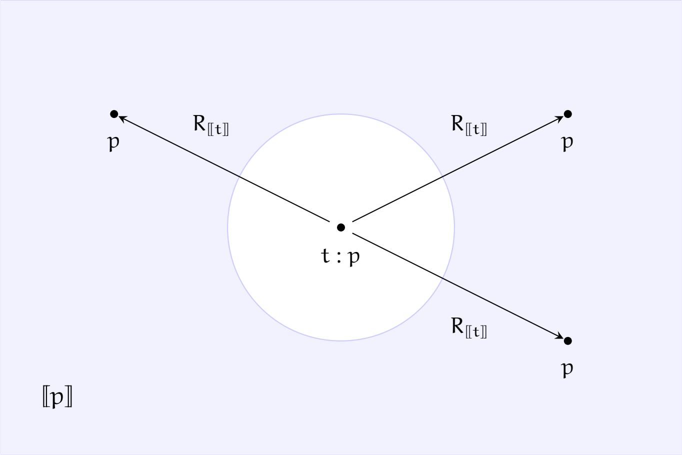

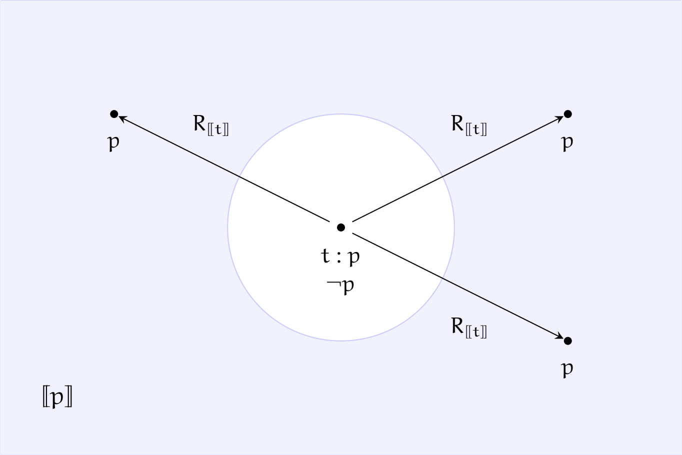
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t : p
¬p
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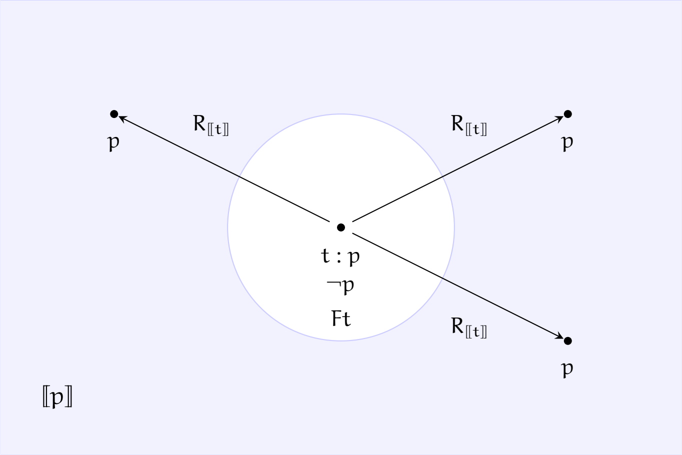

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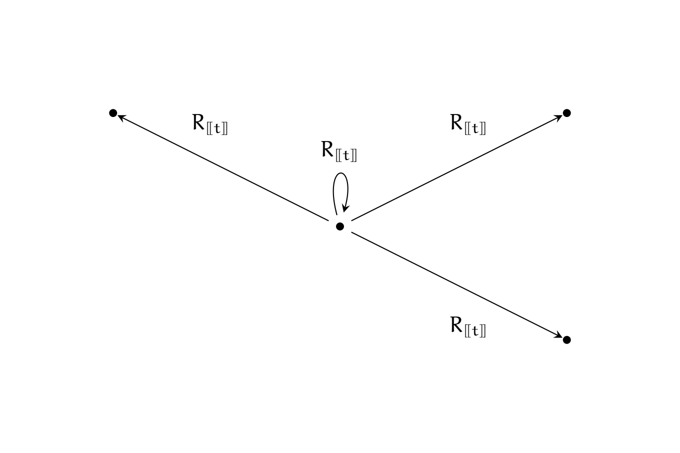

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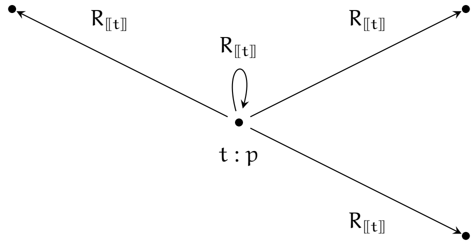

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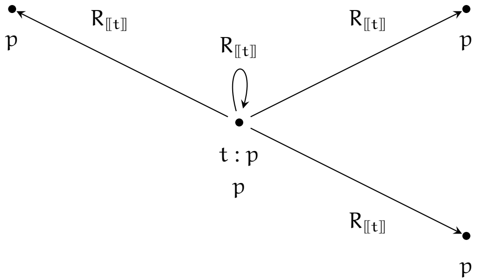

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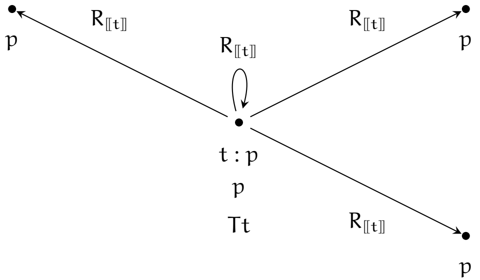

How Truth Behaves

- $\mathfrak{M}, \alpha, w \Vdash \mathrm{D}t$ iff $[\![t]\!]_\alpha \in \mathrm{D}$.

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- $\mathfrak{M}, \alpha, w \Vdash Dt$ iff $[\![t]\!]_\alpha \in D$.
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- $\mathfrak{M}, \alpha, w \Vdash Ft$ iff $[\![t]\!]_\alpha \in D$ and it is not the case that $wR_{[\![t]\!]}w$.

Checking the Axioms

MBA $(\forall x)(\Box(p \supset q) \supset \Box(x : p \supset x : q))$

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(These need more work.)

We need $[\![\ulcorner A \urcorner]\!] \in D$ for each A.

Bradwardine Frames with syntax

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This interaction between truth in the model
and accessibility is not easy to construct.

LIMITS

Starting from scratch

Suppose you have a $\textsc{qs5}^{\forall p}$ model $\mathfrak{M}$, with a denotation for each term $\ulcorner A \urcorner$.

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Transparency is satisfied when $R_{[\![A]\!]}$ is empty.

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Transparency is satisfied when $R_{[\![A]\!]}$ is empty.

We'll start there and add links in the accessibility graph conservatively.

$\mathrm{Br}_0(\mathfrak{M})$ is $\mathfrak{M}$ together with *empty* relations $R_{[\![\ulcorner A \urcorner]\!]}$.

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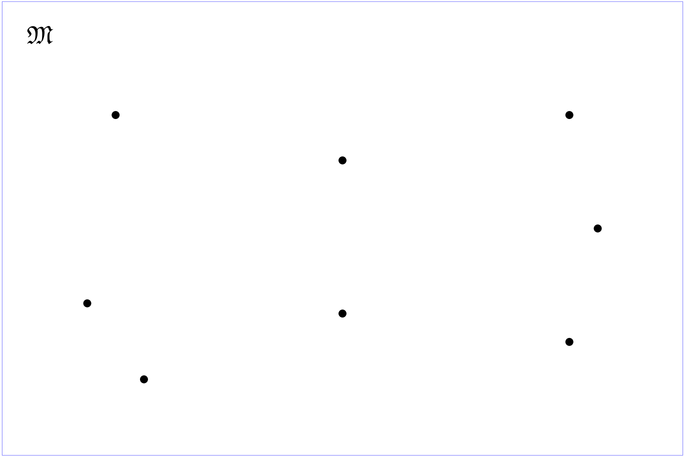
It is the *zero*th Bradwardine model on $\mathfrak{M}$.

Example: $\mathrm{Br}_0(\mathfrak{M})$

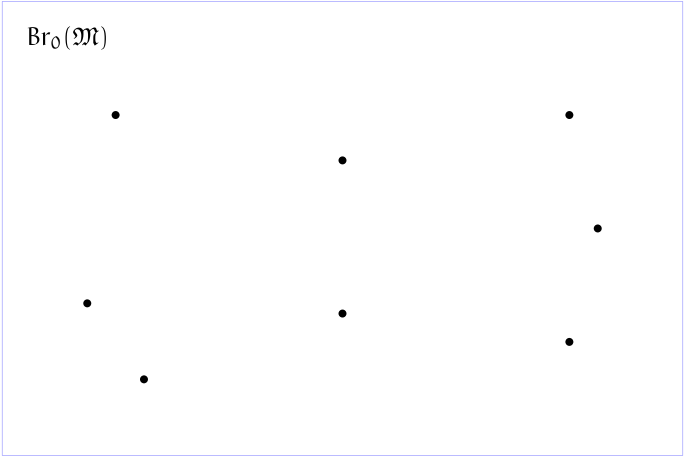


LEGEND

$R_{[[\ulcorner A \urcorner]]}$

$R_{[[\ulcorner B \urcorner]]}$

⋮



Development

A DEVELOPMENT of a model $\mathfrak{S}$
is any model $\mathfrak{S}'$ in which the relations $\mathsf{R}_{[\![\ulcorner A \urcorner]\!]}$
are replaced by $\mathsf{R}'_{[\![\ulcorner A \urcorner]\!]} \supseteq \mathsf{R}_{[\![\ulcorner A \urcorner]\!]}$.

LEGEND

$R_{[[\ulcorner A \urcorner]]}$

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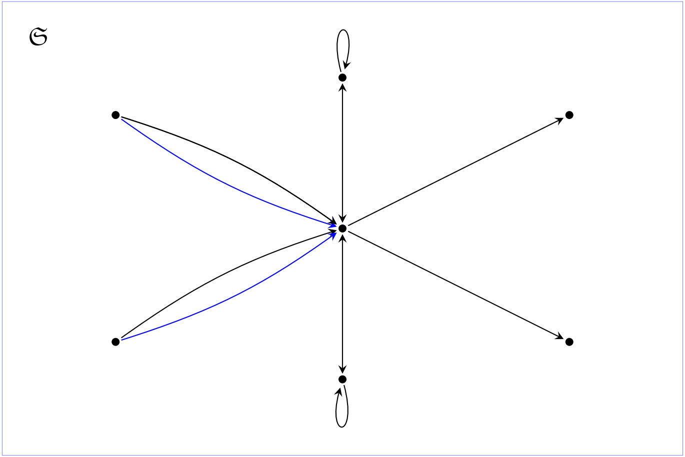
⋮



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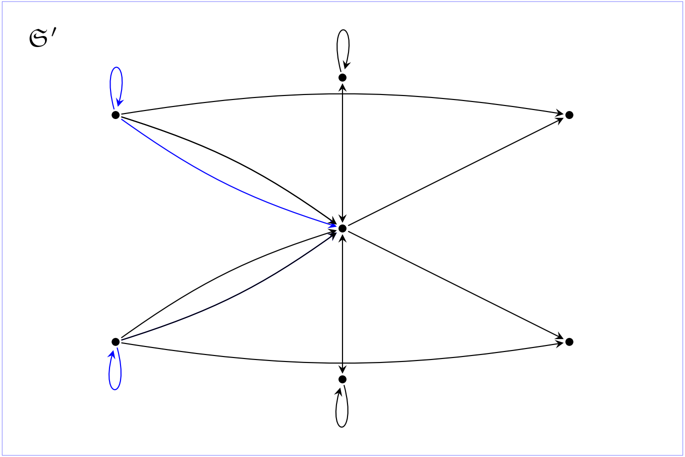


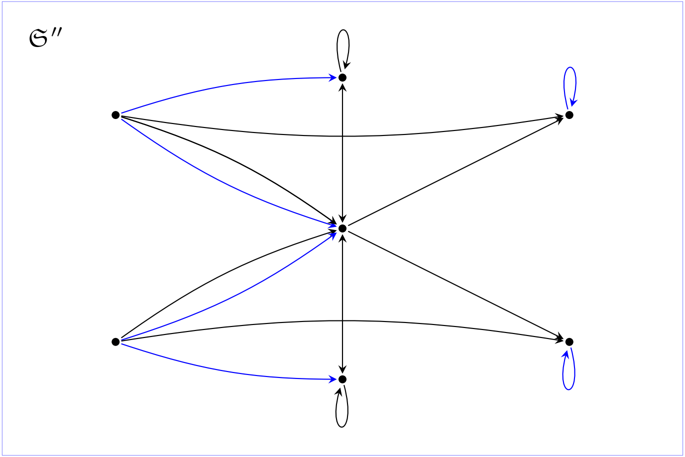
Example: $\mathfrak{S}$ and some of its developments

LEGEND

$R_{[[\ulcorner A \urcorner]]}$

$R_{[[\ulcorner B \urcorner]]}$

$\vdots$



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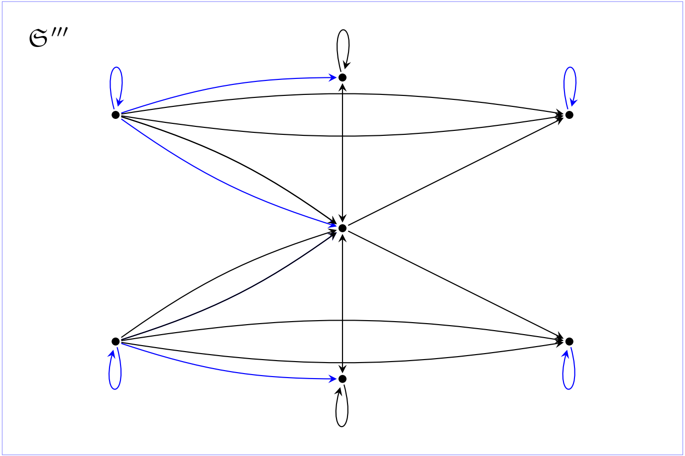
$R_{[[\ulcorner B \urcorner]]}$

⋮



A is said to be SETTLED AT w IN $\mathfrak{M}$ if and only if $(\mathfrak{M}, \alpha, w \Vdash A \text{ iff } \mathfrak{M}', \alpha, w \Vdash A)$ for each development $\mathfrak{M}'$.

Settled sentences

A is said to be SETTLED AT w IN $\mathfrak{M}$ if and only if
$(\mathfrak{M}, \alpha, w \Vdash A$ iff $\mathfrak{M}', \alpha, w \Vdash A)$ for each development $\mathfrak{M}'$.

FACT: Each sentence not containing ":" is settled everywhere in $\mathfrak{M}$.

An arc $wR_{[\![\ulcorner A \urcorner]\!]}v$ is SAFE in $\mathfrak{M}$ iff $\mathfrak{M}', \alpha, v \Vdash A$ in every variant $\mathfrak{M}'$.

Safety

An arc $wR_{[\![\ulcorner A \urcorner]\!]}v$ is SAFE in $\mathfrak{M}$ iff $\mathfrak{M}', \alpha, v \Vdash A$ in every variant $\mathfrak{M}'$.

$\mathfrak{M}$ is safe iff it every arc in $\mathfrak{M}$ is safe.

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$\mathfrak{M}', \alpha, v \Vdash A$ in every variant $\mathfrak{M}'$.

$\mathfrak{M}$ is safe iff it every arc in $\mathfrak{M}$ is safe.

(Safe models don't only validate transparency,
they make it easy to find variants which validate transparency too
— you need only check transparency for *new* arcs.)

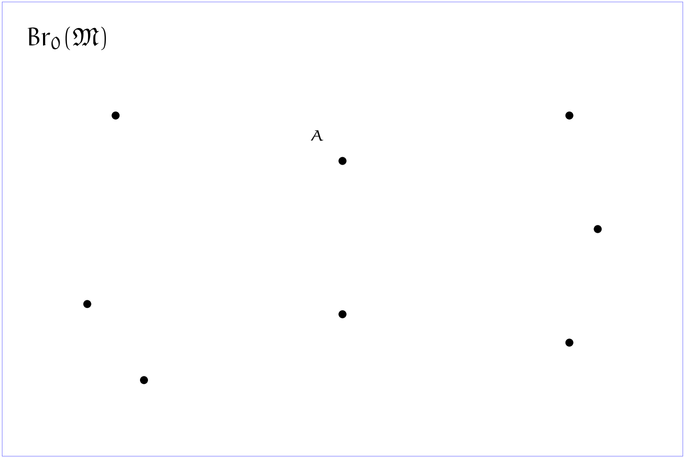
From $Br_0(\mathfrak{M})$ to $Br_1(\mathfrak{M})$

LEGEND

$R_{[[\ulcorner A \urcorner]]}$

$R_{[[\ulcorner B \urcorner]]}$

⋮



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$R_{[[\ulcorner B \urcorner]]}$

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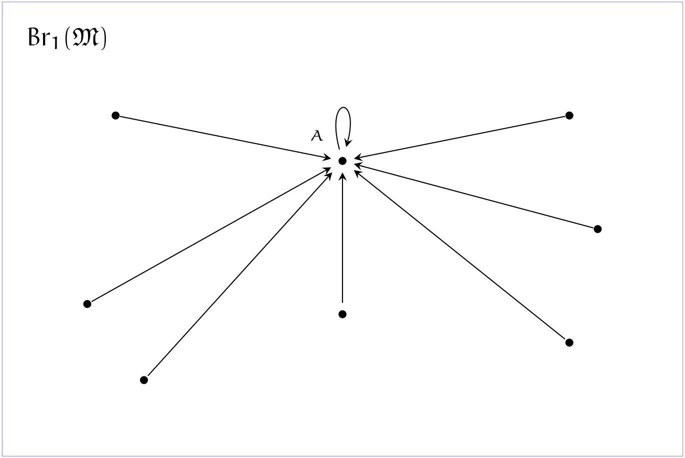

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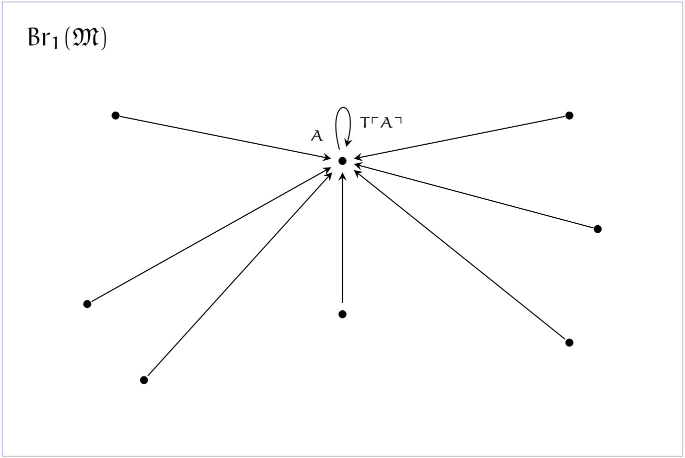

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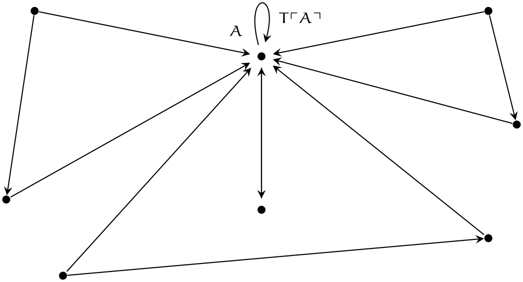
Development of $Br_1(\mathfrak{M})$



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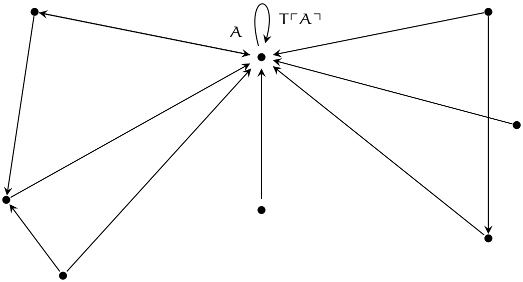

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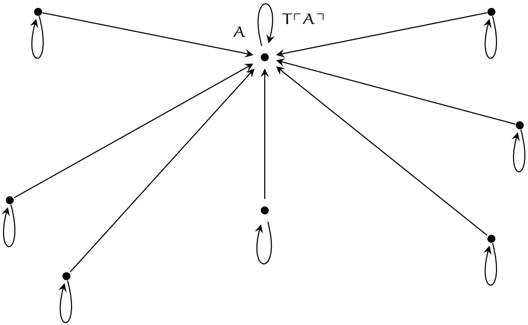

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The variant of $\mathfrak{M}$ with accessibility relations $R'_{[\![\ulcorner A \urcorner]\!]}$ is the said to be the JUMP $\mathfrak{M}'$ of $\mathfrak{M}$.

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$Br_{\alpha+1}(\mathfrak{M})$ is the jump of $Br_{\alpha}(\mathfrak{M})$.

Safe Jumps

FACT: If $\mathfrak{M}$ is safe, then so is its jump $\mathfrak{M}'$.

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$\mathrm{Br}_0(\mathfrak{M})$ is safe.

So, each jump $\mathrm{Br}_n(\mathfrak{M})$ is safe too.

Given a sequence $\mathfrak{M}_\lambda$ $(\lambda < \kappa)$ of safe models such that $\mathfrak{M}_\lambda$ is a variant of $\mathfrak{M}_\epsilon$ whenever $\lambda > \epsilon$,

Limit Stages

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FACT: The limit $\mathfrak{M}_\kappa$ is safe variant of each $\mathfrak{M}_\lambda$ $(\lambda < \kappa)$.

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Let the fixed point be $\mathrm{Br}_*(\mathfrak{M})$.

$\mathrm{Br}_*(\mathfrak{M})$ satisfies transparency

FACT: In $\mathrm{Br}_*(\mathfrak{M})$, $\ulcorner A \urcorner : A$ holds at every point.

$\mathrm{Br}_*(\mathfrak{M})$ satisfies T-intro for grounded sentences

DEFINITION:

- If A is ':'-free, it is grounded at level 0.

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- If A is grounded at α and B is grounded at β, then $\ulcorner A \urcorner : B$ is grounded at $\max(\alpha, \beta)$.
- If A is grounded at α, $T\ulcorner A \urcorner$ and $F\ulcorner A \urcorner$ are grounded at $\alpha + 1$.

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FACT: If A is grounded at stage α, then A is settled at all points in $\mathrm{Br}_\alpha(\mathfrak{M})$.

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FACT: If A is grounded at stage α, then A is settled at all points in $Br_\alpha(\mathfrak{M})$.

COROLLARY: If A is grounded at stage α, then in $Br_*(\mathfrak{M})$, at all points, $A \supset T\ulcorner A \urcorner$.

BEYOND

Leitgeb's Desiderata

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Leitgeb's Desiderata

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7. What about other logics (ternary frames, etc)?