SYMBOLIC LOGIC

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Symbolic logic is sited at intersection of philosophy, mathematics, linguistics and computer science. It deals with the structure of *reasoning*, and the formal features of information. Work in symbolic logic has almost exclusively treated the deductive validity of arguments: those arguments for which it is *impossible* for the premises to be true and the conclusion false. However, techniques from twentieth-century logic have found a place in the study of inductive or probabilistic reasoning, in which premises need not render their conclusions certain.

The historical roots of logic go back to the work of Aristotle, whose *syllogistic* was the standard account — in the Western academy — of the validity of arguments. Syllogistic reasoning treats arguments of a limited form: they have two premises and a single conclusion, and each judgement has form like 'all people are mortal', 'some Australian is poor' or 'no politician is popular'.

The discipline of symbolic exploded in complexity as techniques of algebra were applied to issues of logic in the work of George Boole, Augustus de Morgan, Charles Sanders Peirce and Schröder in the nineteenth century (see Ewald 1996). They applied the techniques of mathematics to represent propositions in arguments *algebraically*, treating validity of arguments like equations in applied mathematics. This tradition survives in the work of contemporary algebraic logicians.

Connections between mathematics and logic developed into the twentieth century, with the work of Gottlob Frege and Bertrand Russell, who used techniques in *logic* to study *mathematics*. Their goals were to use the newfound precision in logical vocabulary to give detailed accounts of the structure of mathematical reasoning, in such a way as to clarify the *definitions* that are used, and to make fully explicit the commitments of mathematical reasoning. Russell and Whitehead's *Principia Mathematica* (Russell and Whitehead, 1912) is the apogee of this project of *logicism*.

With the development of these logical tools came the desire to use them in different fields. In the early part of the twentieth century, the *logical positivsts* attempted to put all of science on a firm foundation by formalising it: by showing how rich theoretical claims bear on the simple observations of experience. The best example of this is the project of Rudolf Carnap, who attempted to show how the logical structure of experience and physical, psychological and social theory could be built up out of an elementary field of perception (Carnap, 1967). This revival of empiricism was made possible by the developments in logic, which allowed a richer repertoire of modes of construction or composition of conceptual content. On an Aristotelian picture, all judgements have a particularly simple form. The new logic of Frege and Russell was able to encompass much more complex kinds of logical structure, and so with it, theorists were able to attempt much more (Coffa 1993).

However, the work of the logical positivists is not the enduring success of the work in logic in the twentieth century. The radical empiricism of the logical positivists failed, not because of external criticism, but because logic itself is more subtle than the positivists had expected. We see this in the work of the two great logicians of the mid-twentieth century. Alfred Tarski's work clarified our view of logic by showing that we can understand logic by means of describing the *language* of logic and the valid arguments by giving an account of *proofs*. On the other hand, we view logic by viewing the *models* of a logical language, and taking a valid argument as one for which there is no model in which the premises are true and the conclusion false. Tarski clarified the notion of a model and he showed how one could rigorously define the notion of *truth* in a language, relative to these models (Tarski 1956). The other great logician of the twentieth century, Kurt Gödel showed that these two views of logic (proof theory and model theory) can agree. He showed that in the standard picture of logic, validity defined with proofs and validity defined by models agrees (see von Heijenhoort, 1967).

Gödel's most famous and most misunderstood result is his *incompleteness* theorem: this result showed that any account of proof for mathematical theories, such as arithmetic, must either be completely intractable (we can never list all of the rules of proof) or be incomplete (not provide an answer for every mathematical proposition in the domain of a theory) or the theory is inconsistent. This result brought the end of the logicist program as applied to mathematics and the other sciences. We cannot view the truths of mathematics as the consequences of a particular theory, and the same holds for the other sciences (see von Heijenhoort, 1967).

Regardless, logic thrives. Proof theory and model theory are rich mathematical traditions, their techniques have been applied to many different domains of reasoning, and connections with linguistics and computer science have strengthened the discipline and brought it new applications.

Logical techniques are tools that may be used whenever it is improtant to understand the structure of the claims we make and the ways they bear upon each other. These tools have been applied in clarifying arguments, analysing reasoning and they feature centrally in the development of allied tools such as statistical reasoning.

One contemporary debate over our understanding of logic also bears on the social sciences. We grant that using languages is a social phenomenon. How does the socially mediated fact of our language use relate to the structure of the information we are able to present with that use of language? Should we understand language as primarily *representational*, with inference valid when what is represented by the premises includes the representation of the conclusion, or should we see the social role of assertion in terms of its *inferential* relations? We may think of assertion as a social practice in which the logical relations of *compatibility* and *reason giving* are fundamental. Once we can speak with each other, my assertions have a bearing on yours, and so, logic finds its home in the social practice of expressing thought in word (Brandom 2000).

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