Proof Theory: Logical and Philosophical Aspects

Greg Restall, Shawn Standefer

Persons in charge: Greg Restall (University of Melbourne), Shawn Standefer (University of Melbourne)

Type: Course

Course outline: Proof theory is the study of formal proof systems. We will focus on sequent systems, which are proof systems in which one reasons about consequence statements, e.g. $A, B \vdash C$. Sequent systems, in contrast to some proof systems, usually contain few axioms but have many rules. In this course, we present an introduction to the proof theory, with a special interest in its interdisciplinary applications. The tools and techniques of proof theory can be applied to systems representing a range of phenomena, including resource-sensitivity, formal grammar, and relevance. The course will be structured as follows.

Lecture 1: We will present the basics of the formalism of sequent systems, give an overview of one of the central theoretical results concerning sequent systems, Gerhard Gentzen's Cut Elimination Theorem, and cover some of its consequences. Time permitting we will survey alternative sequent systems for classical logic.

Lecture 2: We will survey what happens when some of the basic rules of the sequent system are dropped to obtain substructural logics. This will permit us to examine applications of interest to philosophers (relevance logic, theories of truth), computer scientists (linear logic), and linguists (Lambek calculus). Along the way we will highlight connections to combinatory logic.

Lecture 3: Sequent systems are flexible but they have difficulties capturing many modal logics. We provide an overview of the problems of giving sequent systems for modal logics and then turn to generalizations of the basic sequent structure that get around these problems: display logic, hypersequents, and tree-hypersequents. We will present some examples and compare the relative benefits of each sort of generalization.

Lecture 4: We will spend some time with the generalized sequent systems from lecture 3, investigating how they capture different modal logics. In particular, we will focus on hypersequents and how they can be used to provide a uniform treatment of different modal logics. We will also sketch how hypersequent systems can be used to provide proof systems for two-dimensional modal logical concepts, such as actuality and *a priori* knowability.

Lecture 5: We will close out the course by connecting sequent systems to semantics in two different ways. First, we show how to use the basic sequent structure to construct models. Second, we will discuss the philosophical significance of sequent systems. We will discuss the idea that the rules define the connectives, as in inferentialism, and the idea that sequents relate speech acts, such as assertion and denial.

This course will not presuppose much logic. Students should be familiar with classical propositional logic. Prior exposure to either non-classical logic or modal logic will be helpful but will not be necessary. No prior exposure to sequent systems will be presupposed.

Equipment: White board, digital projector

Instructors' teaching experience: Greg Restall has taught logic at all levels of the curriculum from introductory undergraduate logic subjects (interdisciplinary, with colleagues in computer science, engineering, linguistics and mathematics), to advanced graduate seminars, he has taught intensives in substructural logic and proof theory at ESSLLI (Helsinki 2001; Heriot-Watt 2005) and at SELLC (Guangzhou 2010). With Dr. Jen Davoren, he has developed and delivered two Coursera subjects on logic, language and information.

Shawn Standefer has taught non-classical logic at an advanced level in courses aimed at undergraduate philosophy students and has given guest lectures on sequent systems in a logical meta-theory course for undergraduates in philosophy, mathematics, and computer science.