Merely Verbal Disputes and Coordinating on Logical Constants

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1. BACKGROUND

I'm interested in *disagreement*, and I'm interested in *words* and what they mean. In particular, I'm interested in the role that *logic* and *logical concepts* might play in *clarifying* and *managing* disagreement. This includes interest in

- Disagreement between rival accounts of logic
- *Monism* and *Pluralism* anbout logic
- *Ontological* relativity (∃)
- The status of modal vocabulary (◇)

2. A DEFINITION

A man walks rapidly around a tree, while a squirrel moves on the tree trunk. Both face the tree at all times, but the tree trunk stays between them. A group of people are arguing over the question: Does the man go round the squirrel or not?

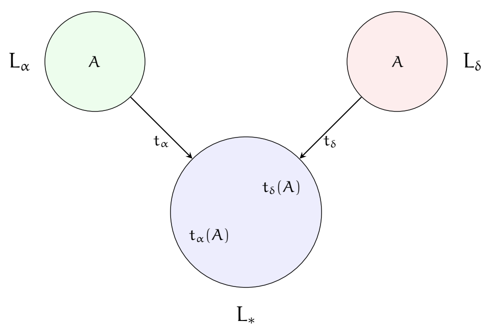
> α: The man *goes round* the squirrel.
> δ: The man *doesn't go round* the squirrel.

Which party is right depends on what you practically mean by 'going round' the squirrel. If you mean passing from the north of him to the east, then to the south, then to the west, and then to the north of him again, obviously the man does go round him, for he occupies these successive positions. But if on the contrary you mean being first in front of him, then on the right of him then behind him, then on his left, and finally in front again, it is quite as obvious that the man fails to go round him … Make the distinction, and there is no occasion for any farther dispute. — William James, *Pragmatism* (1907)

> α: The man *goes round*$_1$ the squirrel.
> δ: The man *doesn't go round*$_2$ the squirrel.

For James, "going round" and "going round" are explicated in terms of prior vocabulary. But even if that is *not possible*, we can resolve a dispute verbally, if α learns δ's term, and δ learns α's.

The general scheme for translation:



Where L_α, L_δ, and L_*, are *languages*, consisting of a *syntax*, a class of *positions* $[X,Y]$ (where X is a collection of things *asserted* and Y, things denied), which may be either *incoherent* (out of bounds; written '$X \vdash Y$') or *coherent* (within bounds; written '$X \nvdash Y$'), satisfying the conditions of IDENTITY ($A \vdash A$); WEAKENING (if $X \vdash Y$ then $X, A \vdash Y$ and $X \vdash A, Y$) and CUT (if $X, A \vdash Y$ and $X \vdash A, Y$ then $X \vdash Y$).

A TRANSLATION $t : L_1 \to L_2$ is a map from one language to another. It may be *incoherence preserving* (if $X \vdash Y$ then $t(X) \vdash t(Y)$) and it may be *coherence preserving* ($X \nvdash Y$ then $t(X) \nvdash t(Y)$), or it may not be. It may also be *compositional* (where the translation of a complex construction is definable from the translations of the components of that construction).

Examples:

- t_α(*going round*) = *going round*$_1$; t_δ(*going round*) = *going round*$_2$.
- *de Morgan* translation: $dm(A \wedge B) = \neg(\neg dm(A) \vee \neg dm(B))$. [*Coherence preserving, incoherence preserving, compositional*]
- Interpreting $L[0,',+,\times]$ into $L[\in]$, modelling the natural numbers as the von Neumann ordinal ω. [The standard translation maps the logical truth $(\forall x)(\exists y)(y = x + 1)$ into the ZF theorem $(\forall x \in \omega)(\exists y \in \omega)(\forall z)(z \in y \equiv (z \in x \vee z = x))$ which is *not* logically true. The translation is not incoherence preserving if incoherence is given by first order logic derivability.

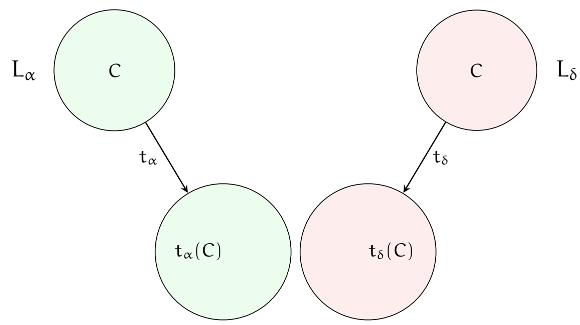
A dispute between a speaker α of language L_α, and δ of language L_δ, over C (where α *asserts* C and δ *denies* C) is said to be RESOLVED BY TRANSLATIONS t_α and t_δ iff there is a language L^* where

- $t_\alpha : L_\alpha \to L^*$, and $t_\delta : L_\delta \to L^*$
- and in L^*, $t_\alpha(C) \nvdash t_\delta(C)$

Given a resolution by translation, there need be no disagreement over C in the shared language L^*, the position $[t_\alpha(C), t_\delta(C)]$ in L^* is coherent.

3. A METHOD…

…for resolving (almost) any dispute by translation: *the way of the undergraduate relativist*.



$$L_{\alpha|\delta} = L_\alpha \sqcup L_\delta$$

Here, $L_{\alpha|\delta}$ is the disjoint union $L_\alpha \sqcup L_\delta$, and $t_\alpha : L_\alpha \to L_{\alpha|\delta}$, $t_\delta : L_\delta \to L_{\alpha|\delta}$ are the obvious injections. For coherence on $L_{\alpha|\delta}$, set

$$(X_\alpha, X_\delta \vdash Y_\alpha, Y_\delta) \text{ iff } (X_\alpha \vdash Y_\alpha) \text{ or } (X_\delta \vdash Y_\delta).$$

This is a coherence relation. The vocabularies *slide past one another*, with no interaction. This translation is *incoherence preserving*, *coherence preserving*, and *compositional*, and it resolves the dispute over C, provided that α was coherent to assert C and δ was coherent to deny it.

4. … AND ITS COST

Nothing α says has any bearing on δ, or *vice versa*.

We have lost conjunction. Even if there is a conjunction connective in L_α and in L_δ, there is none in $L_{\alpha|\delta}$.

Suppose aliens land on earth speaking our languages and familiar with our cultures and tell us that for more complete communication it will be necessary that we increase our vocabulary by the addition of a 1-ary sentence connective V … concerning which we should note immediately that certain restrictions to our familiar inferential practices will need to be imposed. As these Venusian logicians explain, (∧E) will have to be curtailed. Although for purely terrestrial sentences A and B, each of A and B follows from their conjunction A ∧ B, it will not in general be the case that VA follows from VA ∧ B, or that VB follows from A ∧ VB… — Lloyd Humberstone, *The Connectives* §4.34

5. PRESERVATION

Have we got conjunction in L?

$$\frac{X, A, B \vdash Y}{X, A \text{ and } B \vdash Y}[\text{and}\updownarrow]$$

A translation $t : L_1 \to L_2$ is said to be *conjunction preserving* if a conjunction in L_1 is translated by a conjunction in L_2.

6. EXAMPLES

Conjunction: Obviously, there some disagreements can resolved by a disambiguation of different senses of the word 'and' (logicians "and" vs. "and *then*"). However, if

1. '∧' is a conjunction in L_1 and '&' is a conjunction in L_2, and
2. $t_1 : L_1 \to L^*$, and $t_2 : L_2 \to L^*$ are both *conjunction preserving*.

Then '∧' and '&' are *equivalent* in L^*.

That is, in L^*, $A \wedge B \vdash A \,\&\, B$ and $A \,\&\, B \vdash A \wedge B$.

$$\dfrac{\dfrac{A \,\&\, B \vdash A \,\&\, B}{A, B \vdash A \,\&\, B}[\&\uparrow]}{A \wedge B \vdash A \,\&\, B}[\wedge\downarrow] \qquad \dfrac{\dfrac{A \wedge B \vdash A \wedge B}{A, B \vdash A \wedge B}[\wedge\uparrow]}{A \,\&\, B \vdash A \wedge B}[\&\downarrow]$$

Since '∧' and '&' are equivalent, then any merely verbal disagreement betwen $A \wedge B$ and $A' \,\&\, B'$ cannot be explained by an equivocation between '∧' and '&'. The only way to coherently assert $A \wedge B$ and to deny $A' \,\&\, B'$ involves distinguishing A and A' or B and B'.

$$\dfrac{\dfrac{A \vdash A' \qquad \dfrac{B \vdash B' \qquad \dfrac{A' \,\&\, B' \vdash A' \,\&\, B'}{A', B' \vdash A' \,\&\, B'}[\&\uparrow]}{A', B \vdash A' \,\&\, B'}[Cut]}{A, B \vdash A' \,\&\, B'}[Cut]}{A \wedge B \vdash A' \,\&\, B'}[\wedge\downarrow]$$

Negation: It's tempting to analyse the disagreement between the *constructivist* (I assert $\neg\neg p$ and deny p; that's OK, since $\neg\neg p \nvdash p$), and the *classical logician* (I endorse the validity $-{-p} \vdash p$), as merely verbal—disambiguating between different negations.

Have we got negation in L?

$$\frac{X, A \vdash}{X \vdash \neg A}[\neg\updownarrow]$$

A translation $t : L_1 \to L_2$ is said to be *negation preserving* if a negation in L_1 is translated by a negation in L_2. As with conjunction, if

1. '¬' is a conjunction in L_1 and '−' is a conjunction in L_2, and
2. $t_1 : L_1 \to L^*$, and $t_2 : L_2 \to L^*$ are both *negation preserving*.

Then '¬' and '−' are *equivalent* in L^*.

That is, in L^*, $\neg A \vdash -A$ and $-A \vdash \neg A$.

$$\dfrac{\dfrac{-A \vdash -A}{-A, A \vdash}[-\uparrow]}{-A \vdash \neg A}[\neg\downarrow] \qquad \dfrac{\dfrac{\neg A \vdash \neg A}{\neg A, A \vdash}[\neg\uparrow]}{\neg A \vdash -A}[-\downarrow]$$

What options are there for understanding this disagreement, between the classical and constructive logician?

- Disagreement over the consequence relation (*pluralism*).
- The classical thinks the intuitionist is mistaken to take '¬' to be so weak, or the intuitionist thinks that the classical logician is mistaken to take '−' to be so strong.

Existential Quantifier: Obviously, there some disagreements can resolved by a disambiguation of different senses of the quantifiers. Multi-sorted first order logic is well known.

$$\frac{X, A(v) \vdash Y}{X, (\exists x)A(x) \vdash Y}[\exists\updownarrow]$$

When do we have an existential quantifier? (Where v is not free in X and Y.)

However, if

1. '$(\exists_1 x)$' is an existential quantifier in L_1 and '$(\exists_2 x)$' is an existential quantifier in L_2, and
2. $t_1 : L_1 \to L^*$, and $t_2 : L_2 \to L^*$ are both *existential quantifier preserving*.
3. In L^*, some fresh term v is appropriate for both $(\exists_1 x)$ and $(\exists_2 x)$,

Then '$(\exists_1 x)$' and '$(\exists_2 x)$' are *equivalent* in L^*.

$$\dfrac{\dfrac{(\exists_2 x)A(x) \vdash (\exists_2 x)A(x)}{A(v) \vdash (\exists_2 x)A(x)}[\exists_2\uparrow]}{(\exists_1 x)A(x) \vdash (\exists_2 x)A(x)}[\exists_1\downarrow] \qquad \dfrac{\dfrac{(\exists_1 x)A(x) \vdash (\exists_1 x)A(x)}{A(v) \vdash (\exists_1 x)A(x)}[\exists_1\uparrow]}{(\exists_2 x)A(x) \vdash (\exists_1 x)A(x)}[\exists_2\downarrow]$$

This is a *grammatical* condition, it doesn't force agreement on *what exists*.

MONIST:
- $(\forall x)(\forall y)x = y$
- $(\forall y)a = y$
- $a = b$
- Fa, Fb

PLURALIST:
- $(\exists x)(\exists y)x \neq y$
- $(\exists y)a \neq y$
- $a \neq b$
- $Fa, \neg Fb$

Agreement on the grammar is not always obvious (the case of ∧ ≠ ⊃).

Modal Vocabulary: Obviously, there some disagreements can resolved by a disambiguation of different senses of the modal operators. Multi-modal logic is well known.

When do we have an possibility operator?

$$\frac{A \vdash \ | X \vdash Y | \Delta}{X, \Diamond A \vdash Y | \Delta}[\Diamond\updownarrow]$$

However, if

1. '$\Diamond_1$' is an possibility in L_1 and '$\Diamond_2$' is an possibility in L_2, and
2. $t_1 : L_1 \to L^*$, and $t_2 : L_2 \to L^*$ are both *possibilit preserving*.
3. In L^*, a zone is appropriate for $\Diamond_1$ iff it is appropriate for $\Diamond_2$.

Then '$\Diamond_1$' and '$\Diamond_2$' are *equivalent* in L^*.

$$\dfrac{\dfrac{\Diamond_2 A \vdash \Diamond_2 A}{A \vdash \ | \vdash \Diamond_2 A}[\Diamond_2\uparrow]}{\Diamond_1 A \vdash \Diamond_2 A}[\Diamond_1\downarrow] \qquad \dfrac{\dfrac{\Diamond_1 A \vdash \Diamond_1 A}{A \vdash \ | \vdash \Diamond_1 A}[\Diamond_1\uparrow]}{\Diamond_2 A \vdash \Diamond_1 A}[\Diamond_2\downarrow]$$

It's a *dialogical* condition, it doesn't force agreement on *what is possible*.

7. THE UPSHOT

Upshot #1: *The power of keeping some things fixed*. The more you want from a translation, the fewer translations you have, and the fewer ways there are to settle disputes as merely verbal. And the more chance you have to locate that dispute in some particular part of your vocabulary.

Upshot #2: *Defining rules provide fixed points*. It's one thing to think of a logical concept as something satisfying a set of axioms. But that is cheap. Defining rules are more powerful. And defining rules are natural, given the conception of logical constants as topic neutral, and definable in general terms.

Upshot #3: *Generality comes in degrees*. (1) Propositional connectives: *sequents alone* (2) Modals: *hypersequents* (3) Quantifiers: *predicate structure*.

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