Laws of Non-Contradiction, Laws of the Excluded Middle and Logics

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Abstract: There is widespread acknowledgement that the law of non-contradiction is an important logical principle. However, there is less-than-universal agreement on exactly *what* the law amounts to. This unclarity is brought to light by the emergence of paraconsistent logics in which contradictions are tolerated: From the point of view of proofs, not everything need follow from a contradiction — from the point of view of models, there are “worlds” in which contradictions are true. In this sense, the law of non-contradiction is violated in these logics. However, in many paraconsistent logics, statement $\sim(A \wedge \sim A)$ (it is not the case that A and not-A) is still provable. In this sense, the law of non-contradiction is upheld. This paper attempts to clarify the different readings of the law of non-contradiction, in particular taking cues from the tradition of relevant logics. A further guiding principle will be the natural duality between the law of non-contradiction and *rejection* on the one hand and the law of the excluded middle and *acceptance* on the other.

1 *Logics*

Logic is about many different things. One important topic of logic is the relation[1] of *logical consequence*. A logic tells you what follows from what, what arguments are good, and what commitments involve. For the purpose of this paper, I will take a logic to determine a *consequence relation* between premises and conclusions. In particular, a logic will give us a reflexive and transitive relation $\vdash$ on propositions.[2] Not everyone takes logic to be simply about logical consequence — some take the role of logic as primarily determining a class of special propositions, the tautologies. In this paper I will use the notion of logical consequence to clarify the behaviour of the law of non-contradiction, the law of the excluded middle and acceptance and rejection. I will not argue for the primacy of logical consequence for logic — I will merely assume it. However, the flexibility and fruitfulness of this approach to logic will hopefully count towards

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[1]Or *relations* [4].

[2]This view of logic as primarily determining a relation of consequence on propositions is defended in many places. For my “Logical Pluralism” (with JC Beall) [4].

a defence (or perhaps towards the *appeal*) of this view of logic. In what follows we will explore how our commitments to logical consequence are related to our claims we ought accept and claims we ought reject.

Logical consequence constrains acceptance and rejection. What we may rationally accept and what we may rationally reject is connected to logical consequence in a direct way. If I accept a notion of logical consequence, then acceptance and rejection are rationally constrained in the following way: *First*, if $A \vdash B$ and I accept A then I ought accept B too;[3] *Second*, if $A \vdash B$ and I reject B I ought reject A too. Acceptances ought be "closed upwards" under logical consequence and rejections ought to be correspondingly "closed downwards."

If we generalise consequence to relate non-empty *sets* of propositions, taking $\Sigma \vdash \Delta$ to mean that given *all* of the elements of Σ, *some* of the elements of Δ follow, then we constrain acceptance and rejection in a generalised way. If I accept all elements of Σ then I ought accept the disjunction of the elements of Δ.[4] If I reject all elements of Δ, I ought reject the conjunction of the elements of Σ. If conjunction (written '$\wedge$') and disjunction (written '$\vee$') are governed by the usual lattice conditions, then simple results follow constraining acceptance and rejection. Since $A, B \vdash A \wedge B$, it follows that if I accept A and B I ought accept their conjunction. Since, $A \vee B \vdash A, B$ it follows that if I reject A and B I ought to reject their disjunction. The converses are immediate also. $A \wedge B \vdash A$ and $A \wedge B \vdash B$ tell us that if we accept a conjunction we ought accept both conjuncts. Similarly, $A \vdash A \vee B$ and $B \vdash A \vee B$ tell us that if we reject a disjunction we ought reject both disjuncts.

I will not attempt to defend this constraint on rational acceptance and rejection. Any such defence will depend essentially on clarifying both the notion of logical consequence *and* the notion of rationality in play in the account, and this is beyond the scope of a short paper. The task of this paper will be to clarify the consequences of such a constraint for our understanding of the law of non-contradiction, and its dual, the law of the excluded middle.

All proponents of the debate over the interpretation, the defence, or the rejection of the law of non-contradiction and the law of the excluded middle agree that negation connects entailment, acceptance and rejection. Disagreement lies over the form that the connection ought take. One reasonably basic insight, shared by most proponents of classical or non-classical logics, is that negation is "order inverting" in the following sense:

$$\text{If } A \vdash B \text{ then } {\sim}B \vdash {\sim}A \quad (1)$$

This is the inference of *contraposition*. Using our connections between entailment, acceptance and rejection, we reason as follows: If the rejection

[3] Perhaps here I need to also accept that $A \vdash B$, in order to *thereby* be committed to B in virtue of commitment to A. Nothing in this paper will hang on the issue of valid entailments of which we are unaware, for each of the validities we will consider are very easy to demonstrate.

[4] How does this work? At the very least I take it that if, say, Δ is $\{A, B, C\}$ and I ought accept (the disjunction of) $\{A, B, C\}$ and I (rationally) reject A and B then I ought accept the remaining proposition C.

of B brings with it (rationally) the rejection of A, then the acceptance of $\sim B$ brings with it (rationally) the acceptance of $\sim A$. One way to ensure such a connection is to *identify* the acceptance of $\sim A$ with the rejection of A. However, this is not the only way to understand negation. Dialetheists take the rejection of A to (at least sometimes) require something more than the acceptance of $\sim A$, as they sometimes take it that we may sometimes rationally accept both A and $\sim A$. Assuming that we cannot both accept A and reject it at the same time, we have a case where we accept $\sim A$ and do not reject A. Rejection is perhaps more than the acceptance of a negation. Proponents of truth-value gaps take it that acceptance of $\sim A$ might require something more than rejection of A, as they take it that we may sometimes rationally reject both B and $\sim B$. Again, assuming that we cannot accept and reject something at the same time, acceptance is perhaps more than the rejection of a negation.

So much is common ground. There is only so far that one can go using lattice logic with a minimal constraints on negation. In the rest of this paper we will see how endorsing different systems of logical consequence constrains acceptance of contradictions and rejections of excluded middles.

2 *Classical Logics*

Classical propositional consequence is a suitable starting point. Classical logic adds to our basic logic more inferences, such as the law of the excluded middle, in the following form.

$$A \vdash B \vee {\sim}B \tag{2}$$

This tells us that every instance of excluded middle $B \vee {\sim}B$ follows from any proposition whatsoever. This ensures that in the entailment ordering, $B \vee {\sim}B$ is at the top. Endorsing classical logic *very nearly* assures that we are committed to each instance of $B \vee {\sim}B$. That is, if you are committed to classical logic, you are very nearly given compelling reason to accept $B \vee {\sim}B$ for each B. For *any* A, A entails $B \vee {\sim}B$. So if you accept *any* A at all, you have reason to accept $B \vee {\sim}B$. However, this rationally compels us to accept each $B \vee {\sim}B$ only if we antecedently accept some proposition or other. It is consistent with the constraints on acceptance provided by classical consequence that we accept no propositions at all. Then, trivially, our acceptances are closed upward under classically valid inferences.

The situation with the law of non-contradiction is completely dual. The law appears in classical consequence in the following form (which is suggestively called *explosion*):

$$A \wedge {\sim}A \vdash B \tag{3}$$

This tells us that every contradiction $A \wedge {\sim}A$ is at the bottom of the entailment ordering. Endorsing classical logic *very nearly* assures that you are committed to rejecting each contradiction. If you reject any proposition at all, then by (3) you ought reject each contradiction $A \wedge {\sim}A$. However, it is consistent with the constraints on rejection provided by classical consequence that you reject no propositions at all. Then, trivially, your rejections are closed downward under classically valid inferences.

These outlying cases are, of course, exceptions. They are the analogues for acceptance and rejection of the formal result that the empty set and the set of all propositions are both *theories*. (That is, they are sets of formulas closed under logical consequence.) They are undoubtedly theories in this sense, but in a clear sense, they are *trivial*. The empty theory says nothing about the world, and the full theory rules nothing out. There is a simple technique for eliminating such theories from consideration. If we extend consequence further to include *empty* sets Σ and Δ then we eliminate these trivial theories. We read the consequence relation as before. $\Sigma \vdash \Delta$ if and only if any interpretation for each element of Σ is an interpretation for some element of Δ. Classically we will have $\emptyset \vdash B \vee {\sim}B$ (as every interpretation makes $B \vee {\sim}B$ true) and $A \wedge {\sim}A \vdash \emptyset$ (as no interpretation makes $A \wedge {\sim}A$ true). Constraining acceptance and rejection with this more comprehensive consequence relation ensures that the rational agent accept something. For $\emptyset \vdash B \vee {\sim}B$, and since an agent accepts every element of $\emptyset$ (there are none!) the agent ought accept $B \vee {\sim}B$. Similarly, since $A \wedge {\sim}A \vdash \emptyset$ then since an agent rejects every element of $\emptyset$ the agent ought reject $A \wedge {\sim}A$.

By using a more comprehensive consequence relation, with a correspondingly more comprehensive constraint on acceptance and rejection, we have guaranteed that propositions of the form $A \wedge {\sim}A$ ought be rejected and that propositions of the form $B \vee {\sim}B$ ought be accepted. None of the argument has required singling out a primary relation between logic and acceptance (through tautologies) or between logic and rejection (through a set of propositions which logic can determine as worthy of rejection: call them 'inconsistencies'[5]). We get the effect of such special classes of sentences through a more fundamental constraint tying together rational acceptance and rejection with logical consequence — given that the consequence relation is read as relating *sets* of premises and conclusions. The effect of tautologies and inconsistencies is provided by means of empty premise and conclusion sets respectively.

3 Paraconsistent Logics

Not all propositional logics are classical. Not all logics mandate the law of the excluded middle (2) or *explosion* (3). The task of understanding the law of non-contradiction became more pressing with the growing popularity of *paraconsistent* logics [8]. Paraconsistent logics are distinctive in that they do not mandate *explosion* in the form found in (3). Instead, for paraconsistent logics the entailment fails.

$$A \wedge {\sim}A \nvdash B$$

[5]The term is perhaps infelicitous, for the following reason. Here an inconsistency is "not consistent" in the sense that it is not true under any interpretation. According to this definition of the term, a contradiction $A \wedge {\sim}A$ need not be inconsistent in a paraconsistent logic.

The formal details need not detain us here. Suffice to say, in the semantics for these logics there are interpretations in which A and $\sim A$ may both be taken to be true, but in which not *everything* is true. Such interpretations are sufficient to invalidate explosion.

There are many good reasons to endorse a paraconsistent consequence relation [9]. Explosion seems to fail canons of relevance. Circumstances in which contradictions are true seem to be necessary in the evaluation of counterpossible conditionals ("If I squared the circle with ruler and compass then I would be famous" seems true while "If I squared the circle with ruler and compass then Queensland would win the Sheffield Shield in next year" seems false). Commitment to a contradiction does not seem to rationally compel (or even to make rationally *more plausible*) commitment to absolutely everything whatsoever. Finally, some have taken the semantic and set-theoretic paradoxes to furnish convincing proofs that some contradictions are actually *true*. Each reason here seems to motivate a concern for paraconsistent consequence. However, only the last seems to motivate a rejection of the law of non-contradiction in the sense that we are given reason to accept a contradiction, so we are given reason to not reject it.

In many paraconsistent logics the situation with the law of the excluded middle is formally dual to the law of non-contradiction. For example, in the logic of first-degree entailment (the conjunction, disjunction and negation fragment of the relevant logic *R* and its neighbours) we situation with excluded middle is exactly dual. We have

$$A \not\vdash B \vee \sim B$$

as well. Yet many paraconsistent logics, such as *R*, are thought to *include* the law of the excluded middle. In standard presentations for *R* you can *prove* each instance of $B \vee \sim B$! How can this be?

The explanation is not as difficult as it might seem.[6] In a relevant logic like *R* something might be provable without it following from anything and everything. The initial motivation for this is on grounds of relevance. We might be able to prove $B \rightarrow B$ without the tautologous $B \rightarrow B$ following from an arbitrary A. So, in logics like *R* we do not have $A \vdash B \rightarrow B$. However, we do wish say that $B \rightarrow B$ is 'provable' as provable implications are a good record of valid inferences, like $B \rightarrow B$. To this, we must break the link between tautologies (provable propositions in the sense desired here) and those propositions which follow from anything and everything — and in particular, the empty set of premises.[7] We wish to say that logic commends $B \rightarrow B$ as necessary to accept, without saying that $B \rightarrow B$ follows from anything and everything. In a logic such as *R* this is achieved by fiat. The provable propositions are not those at the top of the entailment ordering.[8] More propositions are provable than this. A simple way to rep-

[6]From this point on, I will take the relevant logic *R* as my paradigm case of a paraconsistent logic. Nothing hangs on the logic of implication of *R* here, and many different paraconsistent logics can be substituted without any difference of application.

[7]By structural properties on proofs, if $\emptyset \vdash A$ then it follows straightforwardly that $\Sigma \vdash A$ for any set Σ of premises, at least given the interpretation of set–set consequence given here. For a finer control of premises and conclusions, work on *substructural* logic is relevant [11].

[8]Or equivalently, they are not just the propositions true at every set-up in the frame semantics.

resent this is to add to the language a special proposition t representing the conjunction of all provable propositions.[9] We then have $t \vdash B \rightarrow B$ without also having $A \vdash B \rightarrow B$ for every A. "Logic" dictates that we accept each $B \rightarrow B$ without dictating that such an acceptance follows from any acceptance whatsoever. (As a result, we ought accept t without taking it that $\emptyset \vdash t$.) Accepting *R* and using it to constrain inference in this way involves more than taking rational acceptance to be closed upwards and rational rejection to be closed downwards. Now we *also* take rational acceptance to include t. Without an extra condition such as this we have no way to force the rational acceptance of propositions not at the top of the entailment ordering.

Taking the propositions we ought to accept on the basis of logic alone as those entailed by a *single* proposition t is no loss of generality. First of all, if we ought accept A on the basis of logic, and if A entails B, then we ought accept B on the basis of logic as well. So, the class of propositions to be accepted on the basis of logic is closed upward under entailment. More generally, if we ought accept each element of Σ on the basis of logic and $\Sigma \vdash A$ then we ought accept A, also on the basis of logic. So, if Σ has a conjunction, (call it 't') then we ought accept A on the basis of logic if and only if $t \vdash A$. The only new assumption we have made is that the class Σ of propositions to be accepted on the basis of logic alone has a conjunction.

With these considerations in mind we can proceed to the law of the excluded middle. It ought come as no surprise that in *R* we do not have $A \vdash B \vee {\sim}B$, but we *do* have

$$t \vdash B \vee {\sim}B \tag{4}$$

In accepting *R* we do not hold that $B \vee {\sim}B$ follows from anything and everything, but nonetheless we do hold that we ought accept $B \vee {\sim}B$. Excluded middle is mandated by acceptance of *R* though the acceptance of t, not because each $B \vee {\sim}B$ follows from $\emptyset$.

By dualising, we see that the same sort of treatment is available for the law of non-contradiction. Just as we have a law of the excluded middle just when some proposition t (which we ought to accept) entails $B \vee {\sim}B$ for each B, we also have a law of non-contradiction when some proposition f (which we ought to reject) is entailed by every contradiction.

$$A \wedge {\sim}A \vdash f \tag{5}$$

And *this* may well be provided by a logic. Indeed, (5) is supplied as valid the relevant logic *R*, where f is the negation of t. However, what is typically *not* provided by a logic is guidance on what ought be rejected. The most guidance a logic is usually taken to give is what we have already seen. If some proposition entails *everything* then it is to be rejected if anything is. If a proposition entails the empty conclusion, then it is to be rejected *tout court*. If a logic is paraconsistent, rejecting (3), then the rejection of a contradiction is not so easily read off a consequence relation. More must be given.

[9] For those who prefer the frame semantics, t is true not at every set-up but only at a special class of set-ups, at which all propositions of "logic" are true. Logic dictates that the actual world is one of these set-ups.

In a logic like *R* the guidance to reject contradictions is given by guidance to reject f. Why might we reject f? A plausible reason is that f is $\sim$t, and we have reason to accept t. Given that we ought accept t we ought reject its negation. Now we have travelled in a circle, or at the least, a tight spiral. We have reduced the rejection of an arbitrary contradiction $A \wedge {\sim}A$ to the rejection of the contradiction $t \wedge {\sim}t$. We must reject $t \wedge {\sim}t$ by rejecting its second conjunct $\sim$t, given that we accept its first conjunct t. The reason we have for rejecting arbitrary contradictions, given these considerations, is nothing more than the reason we have for rejecting a particular one, $t \wedge {\sim}t$. Clearly this is not a reason which will find favour with the dialetheist, who takes there to be independent reason for asserting a contradiction. However, for those who have no independent reason to accept any contradiction, the move to accepting a paraconsistent logic gives us no new reasons to accept them. A natural reading of the way that inference in *R* constrains rejection motivates a simple analysis of contradictions according to which they are to be rejected, completely dual to the way that according to *R*, excluded middles are to be accepted.

So far we have seen that a logic like classical logic can mandate the rejection of the law of non-contradiction through the validity $A \wedge {\sim}A \vdash B$. Given a more discriminating logical system in which contradictions need not entail *everything* we may still be given guidance to reject contradictions if they entail something else we ought to reject. In a logic such as *R*, contradictions do entail something which we have good reason to reject. They entail f, the negation of the proposition t which we ought accept. The mere acceptance of a paraconsistent logic like *R* does not *force* us away from rejecting contradictions. The law of non-contradiction can remain in the weaker form, $A \wedge {\sim}A \vdash f$.

On the other hand, nothing in a relevant logic like *R* necessitates the rejection of all contradictions. If we have some independent reason to accept contradictions, then we have independent reason to accept the particular contradiction $t \wedge {\sim}t$. We keep the original reason to accept t, and we now have reason to accept $\sim$t, as it is entailed by the other contradiction we now accept. A paraconsistent logic like *R* is suitable for the dialetheist. We may keep the *R*-edict to close our acceptances upwards and our rejections downward, keep our obligation to accept t, while abandoning the injunction to reject f. Such an approach would not be foreign to the spirit of *R*. But neither would the dual approach: we could well adopt *R*-consequence as a condition on acceptance and rejection, to hold fast to rejecting f while refrain from accepting t, and hence refrain from accepting all propositions of the form $B \vee {\sim}B$. This too would not be foreign to the spirit of *R* as presented here, but it would be foreign to most defenders of *R*, and it is illuminating to consider why. We have presented *R* as primarily a system for determining logical consequence. Traditional presentations of propositional logic, especially those using Hilbert-style proof theories, focus on the special class of logical truths. In *R* the simplest proof theory is the class of propositions entailed by t.[10] This approach to adopting *R* would

[10]This is surely more than a historical accident, given the primacy of implication as an assertoric record of valid inference. However, there is nothing mandatory in *valid inference* which makes implication the only sensible record of validity. We could just as well take

involve taking *R*-consequence as mandatory for regulating acceptance and rejection, while *not* taking *R*-theorems to be rationally mandatory to accept. in particular, an agent guided by this policy would not be thereby obliged to accept each proposition of the form $B \vee {\sim}B$. Such an approach is surely not what Anderson and Belnap intended in *Entailment* [1, 2], but it is not foreign to the enterprise of relevant logic.[11] In just the same way, nothing in the adoption of *R* requires that we reject f and the contradictions which entail it — but it is completely natural to do so. The burden, if there is any at all to be borne, is on the one who fails to reject contradictions to explain why in some case or other, reason to accept t is not reason to reject ${\sim}t$. Nothing in accepting *R*-consequence counts against *that*.[12]

I will end this discussion of forms of the law of non-contradiction with a short analysis of positive forms of the law, as a proposition to be accepted. We have seen the law expressed as the edict "reject $A \wedge {\sim}A$!", which arises in one way or another from a logical consequence relation. What of positive forms of non-contradiction, which enjoin us to accept propositions such as ${\sim}(A \wedge {\sim}A)$. These are present as provable logics such as *R*. In what sense might endorsing of (5) commit one to a positive statement of the law of non-contradiction. Must we accept ${\sim}(A \wedge {\sim}A)$? How can accepting this follow from (5)? Here is one way: a contraposition of (5) gives

$${\sim}f \vdash {\sim}(A \wedge {\sim}A)$$

In *R*, ${\sim}f$ is equivalent to t (as ${\sim}{\sim}A$ is equivalent to A.) So we are a small step away from accepting ${\sim}(A \wedge {\sim}A)$. For this, we need only accept ${\sim}f$. Even if we reject the inference from ${\sim}{\sim}t$ to t in general, we may have reason to accept it in this case. If we reject f, then surely we ought accept its negation. Or should we? If I take f to be *neither true nor false* then I may accept neither f nor ${\sim}f$. In this case, accepting *this* form of the law of non-contradiction seems indeed to depend on the law of the excluded middle. This makes sense. If we *reject* $A \wedge {\sim}A$, this only makes one *accept* ${\sim}(A \wedge {\sim}A)$ given a version of the law of the excluded middle. In *R*, given that we accept t we ought accept ${\sim}(A \wedge {\sim}A)$ also.[13] This explains how the *R*-proponent who accepts closure under *R*-consequence, and who accepts t need not also accept the law of non-contradiction on the form of rejecting f — but will still happily assent to all instances of ${\sim}(A \wedge {\sim}A)$. This is no

subtraction as primary (read '$B - A$' as "B without A") and instead of taking $t \vdash A \rightarrow B$ as our asserted record of $A \vdash B$, take $B - A \vdash f$ as our *denied* record of the same entailment. We reject $B - A$ on the basis of logic alone in just the same way as we accept $A \rightarrow B$ on that basis. We cannot establish a priority for assertion on merely formal grounds. An "axiomatisation" of *R* on the basis of the propositions we ought *reject* on the basis of logic alone (those which entail f) is just as straightforward as its traditional axiomatisation. (Note that $B - A$ is equivalent in *R* to ${\sim}(A \rightarrow B)$.)

[11] It is the analogue for agents of the formal result that not all *R*-theories need be *regular* in the sense of containing all *R*-theorems. Or simply, t is not a member of all *R*-theories.

[12] There is more to this issue than I have time or space to address here. Priest has ingeniously argued that while accepting a paraconsistent logic does not rationally *force* one to be open to accept contradictions, it is the first step down a swift slippery slope in that direction [7]. I do not think that Priest's argument to this conclusion is compelling as it stands, but I cannot address it here. Beall and I have a paper which addresses this issue at some length [3].

[13] The argument can go directly from $t \vdash A \vee {\sim}A$ to $t \vdash {\sim}(A \wedge {\sim}A)$ by de Morgan's laws as well. All de Morgan laws are available in *R*. This also makes the connection between $A \vee {\sim}A$ and ${\sim}(A \wedge {\sim}A)$ explicit.

more inconsistent than accepting the particular contradictions which set this agent down this route. The only trouble for this believer is the multiplication of contradictions. If she accepts $A \wedge {\sim}A$, and also accepts t then she ought accept ${\sim}(A \wedge {\sim}A)$ also — another contradiction with her earlier acceptance of $A \wedge {\sim}A$. The positive form of the law of non-contradiction, as an acceptance of ${\sim}(A \wedge {\sim}A)$ does not obligate the agent to reject contradictions independently of her obligation to reject f. If she does without that obligation, then merely accepting ${\sim}(A \wedge {\sim}A)$ will do no good. At the risk of belabouring the duality, the situation exactly parallel to the case of the agent who rejects some instances of $B \vee {\sim}B$ (and thereby rejects t). The mere fact that she also may reject ${\sim}(B \vee {\sim}B)$ is not going to obligate her to accept $B \vee {\sim}B$. If she rejects both t and ${\sim}t$, she is not going to be obliged to accept $B \vee {\sim}B$ simply because she rejects ${\sim}(B \vee {\sim}B)$. If excluded middles have no purchase in general, then there is no general reason to accept that either $B \vee {\sim}B$ or its negation ${\sim}(B \vee {\sim}B)$ is true. If there are truth-value gaps with B, we may well expect there to be truth-value gaps with $B \vee {\sim}B$ too. If there are truth-value gluts with A we just as well might expect truth-value gluts with $A \wedge {\sim}A$ too. In the absence of rejection mechanisms in general, accepting ${\sim}(A \wedge {\sim}A)$ will not oblige one to reject contradictions.

4 Disjunctive Syllogism and its Dual

I will end by discussing a close analogue to the law of non-contradiction — *disjunctive syllogism*. I have written elsewhere about one general approach to disjunctive syllogism [4, 10, 11], and I will not repeat that analysis here. In those papers I defended both *R*-consequence and classical consequence as appropriate accounts of valid inference, and I took the difference in their treatment of disjunctive syllogism as indicative of our need to attend to both canons of validity. In those discussions, I did not attend to the particularities of acceptance and rejection, and neither did I attend to the resources available in *R*-consequence, supplemented with t and f. So let us start: Disjunctive syllogism, in the following form

$$A \vee B, {\sim}A \vdash B \qquad (6)$$

is classically valid, but not valid in *R*. Here is the closest that *R* can furnish us with an inference in its vicinity:

$$A \vee B, {\sim}A \vdash B, f \qquad (7)$$

If I have reason to accept both $A \vee B$ and ${\sim}A$, then I ought accept the disjunction of B and f. (The inference can go through the disjunction of $B \wedge {\sim}A$ and $A \wedge {\sim}A$ (by distribution), then the inference from $A \wedge {\sim}A$ to f, and by discarding the conjoined ${\sim}A$ in the first conclusion.) What I can do given such a disjunctive acceptance is a matter of further discussion. The practice of simply dropping the disjoined f or saying it *sotto voce* is discussed, and discarded as unsatisfactory, in Belnap and Dunn's comprehensive account of the issue [5].

In keeping the spirit of this paper, I will examine what we ought say about (7) and its constraint on acceptance and rejection in terms of its dual inference form. The dual of (6) is also invalid in *R*

$$A \vdash {\sim}B, A \wedge B \tag{8}$$

but its companion, dual to (7) with a conjoined t as premise, is valid in *R*.

$$t, A \vdash {\sim}B, A \wedge B \tag{9}$$

As far as I have been able to ascertain, this inference form is not discussed in connection with disjunctive syllogism.[14] It is illuminating to consider how *R*-obligations on acceptance and rejection constrain us in each case. We will start with (9), as it is less worked over.

The relevant direction of constraint with parallels for disjunctive syllogism is the *R*-constraint of *rejection*. If I reject both $A \wedge B$ and ${\sim}B$ then by *R*-consequence, I ought reject the conjunction of t and A. Given that I ought not reject t, it follows that I ought reject A. Here we have a straightforward recapture of the "classical" inference pattern (8) as far as rejection preservation is concerned. Given that I ought not reject t (as a full-blown acceptance of *R* would have us eschew) then if I reject $A \wedge B$ and ${\sim}B$, I ought reject A too. Why is this? Well, given that I accept t, I accept $B \vee {\sim}B$. I reject ${\sim}B$ so I ought accept B. (I cannot accept $B \vee {\sim}B$ while rejecting both disjuncts.) But now, I reject $A \wedge B$ but I accept one conjunct B. I ought thereby reject A, lest I accept the entire conjunction.[15]

Now let's see the analogue for (7). Given that I accept $A \vee B$ and I accept ${\sim}A$, what ought the full-blooded *R*-proponent do? Given that I ought reject f, but I ought accept $B \vee f$, I ought accept B, for I ought not accept a disjunction and reject both disjuncts. The rejection of f mandated by *R* provides enough transmission of acceptance to give the effect of disjunctive syllogism. If I do not have reason to reject f, then the situation is also straightforward. If I do not reject f then the inference gives me no extra reason to accept B. The case is direct in the hypothetical situation in which one accepts A and ${\sim}A$ for the sake of the argument. There is no sense that

[14]The closest discussion to this one in the literature that I know of is Graham Priest's in section 8.3 of *In Contradiction* [6].

[15]As for constraining *acceptance*, (9) tells us that if I accept t and I accept A, then I ought accept the disjunction of ${\sim}B$ and $A \wedge B$. But I ought accept t anyway, so if I accept A then since I ought accept t, I ought accept both A and t, and thus, accept the disjunction of ${\sim}B$ and $A \wedge B$. So, given that we ought accept t, we get the effect of (8) without accepting it as valid, in just the same way as the obligation to accept t rationally obligates our acceptance of $B \vee {\sim}B$ without necessitating our acceptance of (3). The weaker (5) suffices.

But in the case of (9) the argument utilises an important extra feature: the fact that the acceptance of A *survives* the acceptance of t. We can only reason as we did, from the acceptance of A to the obligation to accept the disjunction of ${\sim}B$ and $A \wedge B$, only if the obligation to accept t, when fulfilled, keeps the acceptance of A in play. For only when we accept *both* t and A will (9) take effect and the obligation of to accept the conclusion follow. If once we accept t the acceptance of A disappears, (9) gains no grip. This seems possible. Suppose A is ${\sim}t$, and I accept ${\sim}t$, and that I *don't* also accept t. Here I fall foul of what *R* enjoins us to accept and reject. Nothing I have said so far tells us that if I have failed to live up to one *R*-obligation the others fail to apply. If I accept ${\sim}t$, then here is how *R* constrains my acceptances: I ought accept t, and if I accept $t \wedge {\sim}t$, I ought accept the disjunction ${\sim}B$ or ${\sim}t \wedge B$. Now if I have no inclination to accept $t \wedge {\sim}t$ then the second obligation does not apply, and we are left with the obligation to accept t. If a stronger case could be made, to the effect that I *always* accept t, so that once I accept ${\sim}t$ I *thereby* am committed to $t \wedge {\sim}t$, the second condition applies, and any riders about disappearing acceptances do not arise.

this acceptance *thereby* mandates you to accept B. No, if I have accepted both A and ~A, then although $A \vee B$ follows, and as a result, so does $B \vee f$, this gives us no reason to accept B — for the assumption of both A and ~A brought along with it an assumption of f, and *this* is how we were lead to $B \vee f$, not an inference to B alone.

5 Conclusion

We have seen that considering a logic as a consequence relation makes the parallels between laws of non-contradiction and laws of the excluded middle striking. In classical logic, the laws are strong. Contradictions are at the bottom of the entailment ordering, and excluded middles are at the top. In logics such as *R* the situation is more subtle. Contradictions need not entail everything, and excluded middles need not be entailed by everything. Yet, just as it is natural to accept excluded middles, on the basis of *R*-reasoning, it is natural to reject contradictions on the basis of the very same reasoning. A paraconsistent logic such as *R* makes the way *open* for the acceptance of contradictions, and controls the consequences of such acceptances, but it does not make them mandatory. There are many different laws of non-contradiction, and, and some are present, even paraconsistent logics such as *R*.

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