Assertions, Denials Questions, Answers & the Common Ground

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My ***Aim***

To better understand the speech acts of *assertion* and *denial*, their relationships to *other* speech acts, and connections between speech acts and logical notions, including the Gentzen's sequent calculus.

My Prompt

I want to revisit some themes
(and expand on some of the claims)
from my 2005 paper
"Multiple Conclusions."

The behaviour of two kinds of speech acts:

(1) *polar* (yes/no) *questions*,

and (2) *justification requests*.

My ***Plan***

Assertion and Denial

Polar Questions

Positions and Rules

Justification Requests

ASSERTION AND DENIAL

$$X \succ Y$$

Don't *assert* each member of X
and *deny* each member of Y.

Structural Rules

$$X, A \succ A, Y \; {\scriptstyle Id} \qquad \frac{X \succ A, Y \quad X, A \succ Y}{X \succ Y} \, {\scriptstyle Cut}$$

These rules govern assertion and denial *as such*.

Defining Rules for Logical Concepts

$$\begin{array}{c} X, A, B \succ Y \\ \hline\hline X, A \land B \succ Y \end{array}\ {\land}\mathit{Df} \qquad \begin{array}{c} X \succ A, B, Y \\ \hline\hline X \succ A \lor B, Y \end{array}\ {\lor}\mathit{Df} \qquad \begin{array}{c} X \succ A, Y \\ \hline\hline X, \neg A \succ Y \end{array}\ {\neg}\mathit{Df} \qquad \begin{array}{c} X, A \succ B, Y \\ \hline\hline X \succ A \to B, Y \end{array}\ {\to}\mathit{Df}$$

$$\begin{array}{c} X \succ A(n), Y \\ \hline\hline X \succ \forall x A(x), Y \end{array}\ {\forall}\mathit{Df} \qquad \begin{array}{c} X, A(n) \succ Y \\ \hline\hline X, \exists x A(x) \succ Y \end{array}\ {\exists}\mathit{Df} \qquad \begin{array}{c} X, Fa \succ Fb, Y \quad X, Fb \succ Fa, Y \\ \hline\hline X \succ a = b, Y \end{array}\ {=}\mathit{Df}$$

Terms & conditions: the singular term n (in $\forall/\exists\mathit{Df}$) and the predicate F (in $=\mathit{Df}$) do not appear below the line in those rules.

These rules can be understood as *definitions* of the concepts they introduce (below the double line).

See (Scott 1974; Došen 1980, 1989; Restall 2019).

… I was wading into a pre-existing literature about assertion. A *very large* literature.

It is fruitful to think of assertion
as an act governed by *norms*.

*For **me**: Production Norms*

Aim to say what is *true*!

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Only say what you *know*!

*For **me**: Production Norms*

Aim to say what is *true*!

Only say what you *know*!

Be prepared to *back it up* when requested!

The hearer is entitled to re-assert.

*For **you**: Consumption Norms*

The hearer is entitled to re-assert.

You can refer back to the asserter
to *vouch for* the assertion.

To assert is to bid for the content asserted
to be added to the COMMON GROUND,
the body of information that
we (together) take for granted.

Stalnaker on Common Ground

To presuppose something is to take it for granted, or at least to act as if one takes it for granted, as background information as common ground among the participants in the conversation. What is most distinctive about this propositional attitude is that it is a social or public attitude: one presupposes that ϕ only if one presupposes that others presuppose it as well.

— "Common Ground" *L&P* (2002)

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This does not help distinguish *denial* from *retraction*, or from other speech acts.

… by examining polar questions,
and their answers,
in the light of our background
interest in assertion and its norms.

POLAR QUESTIONS

This is a distinct speech act
with its own norms.

Is it the case that p?

This is a distinct speech act
with its own norms.

It raises an *issue*.

*There are two ways to **settle** the issue*

Yes

*There are two ways to **settle** the issue*

Yes *No*

If I say *yes* and you say *no*
to some polar question p?,
then we DISAGREE.

That is, we take *different* positions on p.

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to some polar question p?,
then we DISAGREE.

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There is no *shared* position
incorporating both of our answers.

Other responses, like

Other responses, like

maybe

Other responses, like

maybe · *I don't know*

Other responses don't settle the issue

Other responses, like

maybe · *I don't know* · *I think so*

Other responses, like

maybe · *I don't know* · *I think so*

are acceptable responses to p?,

Other responses, like

maybe · *I don't know* · *I think so*

are acceptable responses to p?,

but they don't answer the question.

They don't settle the issue of p.

Settling answers are assertions

A *yes* or a *no* to p? counts as an assertion.

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A *yes* or a *no* to p? counts as an assertion.

(Either answer is governed by all of the assertion norms we've seen.)

Presumably $\neg p$.

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However, I prefer to think of
a *yes* to p? as ruling p *in*,
and a *no* to p? as ruling p *out*.

What does a "no" to p? ***assert***?

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This way, we can distinguish practices
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(Nothing important *here* hangs on this distinction.)

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a pair of sets of sentences

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- We have ruled *in* everything in X, the POSITIVE COMMON GROUND.
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Common Ground

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- We have ruled *in* everything in X, the POSITIVE COMMON GROUND.
- We have ruled *out* everything in Y, the NEGATIVE COMMON GROUND.

Think of this as part of the *conversational scoreboard*.
There are also our public *individual* commitments,
the questions under discussion, and much more.

Denial and Retraction

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ELOISE: No, he is in the kitchen.

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ELOISE: Maybe. He's *either* in the kitchen *or* the study.

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ABELARD: Is Astralabe in the study?

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WEAK DENIAL

ABELARD: Is Astralabe in the study?

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PARTIAL ANSWER

- To *strongly deny* p is to bid to add p to the *negative common ground*.

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- To *weakly deny* p is to *block* the addition of p to the *positive common ground*, or to bid for its *retraction* if it is already there.

Strong and Weak Denial, and the Common Ground

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- Strong *or* weak denials of p are appropriate responses to an assertion of p, because the assertion of p is a bid to add p to the positive common ground.
- A strong denial of p is one way to settle the question p? — to settle it negatively.
- A weak denial of p is not generally an appropriate response to the polar question p?, as the polar question does not place p in the positive common ground, and the question is inappropriate if p is already in the positive common ground, so there is no p to block or retract.

- STRONG DENIAL: add to the negative common ground.

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Strong and Weak Denials, and Strong and Weak ***Assertions***

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- STRONG ASSERTION: add to the positive common ground.
- WEAK DENIAL: retract (or block) from the positive common ground.
- WEAK ASSERTION: retract (or block) from the negative common ground. — "Perhaps p."

That's one way to understand the relationship between assertion and denial, and how to distinguish strong denial from other negative speech acts.

One Consequence

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ELOISE: No. The interior angles of *this* triangle add up to 180°. Can you prove the general case?

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ELOISE: No. The interior angles of *this* triangle add up to 180°. Can you prove the general case?

Eloise blocks from the common ground (weakly denies)
a *logical consequence* of the common ground (the axioms of geometry),
for the same kind of reason we accept for other weak denials.

This would be impossible if the common ground was simply a set of worlds.

If $X \succ Y$ is derivable,
then it's out of bounds
to *strongly assert* each member of X
and *strongly deny* each member of Y.

Logical Consequence and Strong or Weak Denial

If $X \succ Y$ is derivable,
then it's out of bounds
to *strongly assert* each member of X
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But this example shows that
it *need not* be out of bounds to
strongly assert each member of X
and *weakly deny* each member of Y.

Any position $[X, A : A, Y]$
in which A has been
strongly asserted and
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If $X \succ Y$ is not derivable then $[X : Y]$ is *available*.

A Word on ***Cut***

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(After all, isn't the issue A? thereby *implicitly* settled by $[X : Y]$?)

POSITIONS AND RULES

Defining Rules

$$\frac{X, A, B \succ Y}{X, A \land B \succ Y}\ \land Df \qquad \frac{X \succ A, B, Y}{X \succ A \lor B, Y}\ \lor Df$$

$$\frac{X \succ A, Y}{X, \neg A \succ Y}\ \neg Df \qquad \frac{X, A \succ B, Y}{X \succ A \to B, Y}\ \to Df$$

$$\frac{X, A, B \succ Y}{X, A \land B \succ Y}\ \land Df \qquad \frac{X \succ A, B, Y}{X \succ A \lor B, Y}\ \lor Df$$

$$\frac{X \succ A, Y}{X, \neg A \succ Y}\ \neg Df \qquad \frac{X, A \succ B, Y}{X \succ A \to B, Y}\ {\to} Df$$

These are kinds of *definitions*, showing how to treat assertions or denials of the *defined* concept in terms of the assertions or denials of their components.

Derivations

$$\cfrac{\cfrac{\neg p \succ \neg p}{\succ p, \neg p}\;{\scriptstyle \neg Df}}{\succ p \lor \neg p}\;{\scriptstyle \lor Df}$$

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$$\dfrac{\dfrac{\neg p \succ \neg p}{\succ p, \neg p}\,\neg Df}{\succ p \lor \neg p}\,\lor Df \qquad \dfrac{\dfrac{p \succ p}{p, \neg p \succ}\,\neg Df}{p \land \neg p \succ}\,\land Df$$

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$$\dfrac{\dfrac{\neg p \succ \neg p}{\succ p, \neg p}\,{\scriptstyle \neg Df}}{\succ p \vee \neg p}\,{\scriptstyle \vee Df} \qquad \dfrac{\dfrac{p \succ p}{p, \neg p \succ}\,{\scriptstyle \neg Df}}{p \wedge \neg p \succ}\,{\scriptstyle \wedge Df}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{p, q \vee r \succ p \wedge q, q \vee r}{p, q \vee r \succ p \wedge q, r, q}\,{\scriptstyle \vee Df} \quad \dfrac{p \wedge q, q \vee r \succ p \wedge q, r}{q, p, q \vee r \succ p \wedge q, r}\,{\scriptstyle \wedge Df}}{p, q \vee r \succ p \wedge q, r}\,{\scriptstyle Cut}}{p, q \vee r \succ (p \wedge q) \vee r}\,{\scriptstyle \vee Df}}{p \wedge (q \vee r) \succ (p \wedge q) \vee r}\,{\scriptstyle \wedge Df}$$

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Sequent Derivations aren't exactly ***Proofs***

- They don't have the same *shape* as proofs.
- (Where is the *conclusion* in $p \lor q \succ p, q$?)
- A endsequent $X \succ A$ doesn't tell you to *infer* A *from* X — it merely tells you to not assert all members of X and deny A.

Let's make this problem ***sharp***

"Well, now, let's take a little bit of the argument in that First Proposition—just *two* steps, and the conclusion drawn from them. Kindly enter them in your note-book. And in order to refer to them conveniently, let's call them *A*, *B*, and *Z*:—

(*A*) Things that are equal to the same are equal to each other.

(*B*) The two sides of this Triangle are things that are equal to the same.

(*Z*) The two sides of this Triangle are equal to each other.

Readers of Euclid will grant, I suppose, that *Z* follows logically from *A* and *B*, so that any one who accepts *A* and *B* as true, *must* accept *Z* as true?"

"Undoubtedly! The youngest child in a High School—as soon as High Schools are invented, which will not be till some two thousand years later—will grant *that*."

"And if some reader had *not* yet accepted *A* and *B* as true, he might still accept the *sequence* as a *valid* one, I suppose?"

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The Tortoise never asserts A and A $\to$ Z while denying Z, but he doesn't accept A and A $\to$ Z *as a reason* for Z.

JUSTIFICATION REQUESTS

What is a justification request?

ABELARD: Astralabe is in the kitchen.

ELOISE: *Really*?

ABELARD: I saw him there five minutes ago.

ELOISE: OK.

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ELOISE: *Are you sure*? He's been in the study with me for the last half hour.

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ABELARD: Astralabe is in the kitchen.
ELOISE: *Really*?
ABELARD: I saw him there five minutes ago.
ELOISE: Yes, but he was in the study two minutes ago.

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That's a JUSTIFICATION REQUEST.

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This reason is another assertion [or denial] which must be *granted*, (added to the common ground) in order for the request to be met.

Granting the given reason is *necessary* but not *sufficient* for satisfying the justification request.

Definitions and Justification Requests

ACHILLES So … this is an *equilateral* triangle.

TORTOISE I'm sorry, I don't follow, my heroic friend. I've not heard that word before: what does '*equilateral*' mean?

ACHILLES Oh, that's easy to explain. '*Equilateral*' means having sides of the same length. An *equilateral* triangle is a triangle with all three sides the same length.

TORTOISE OK. That sounds good. You may continue with your reasoning.

ACHILLES Well, as I was saying, the sides of this triangle are all one cubit in length, so it is an equilateral triangle.

TORTOISE Perhaps you will forgive me, Achilles, but I still don't follow. I grant to you that the sides of this triangle all have the same length. I fail to see, however, that it *follows* that it is an equilateral triangle. Could you explain why it is?

If I accept the definition $A =_{df} B$,
then I should accept granting A as meeting
a justification request for the assertion of B
and ruling out A as meeting a justification
request for B's denial and *vice versa*.

A failure to accept this is a sign
that I have not mastered the definition.

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It is a mistake to grant A and grant B and to look for something more to discharge a justification request for an assertion of $A \land B$, if you take $\land Df$ as a *definition*.

Justification Requests and Defining Rules

$$\frac{X, A \succ B, Y}{X \succ A \to B, Y}\ {\to}Df$$

$$\frac{X, A \succ B, Y}{\overline{X \succ A \to B, Y}}\ {\to}Df$$

It is a mistake to rule A in and rule B out and to look for something more to discharge a justification request for a denial of $A \to$ B if you accept $\to$*Df* as a definition.

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Consider this *focussed* derivation:

$$\dfrac{A \to Z \succ \boxed{A \to Z}}{A \to Z, A \succ \boxed{Z}}\ {\to}Df$$

- Read the *premise* as telling us that in a position in which $A \to Z$ is already ruled in, we have an answer to the justification request for $A \to Z$'s assertion.

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- Read the *premise* as telling us that in a position in which $A \to Z$ is already ruled in, we have an answer to the justification request for $A \to Z$'s assertion.
- Then applying $\to$*Df* we see why we have an answer to the request concerning Z's assertion, in a context in which $A \to Z$ and A have both been ruled in. (In granting $A \to Z$ and A we have settled Z positively. Its denial is ruled out, since to assert A and deny Z amounts to denying $A \to Z$.)

SLOGAN: A *derivation* of $X \succ \boxed{A}, Y$ shows us how to meet a justification request for the assertion of A in any available position extending $[X : Y]$.

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(Note: it's the *derivation* that shows how to meet the justification request, not the mere validity of the sequent.)

Details

See the handout for details on
derivations with *focus*
and meeting *justification requests*.

Answers!

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- Since the common ground from which the request is met can contain assertions and denials, we derive sequents of the form $X \succ A, Y$ and $X, A \succ Y$.
- The making of an *inference* is a (possibly preemptive) answer to a justification request.
- A derivation of a sequent $X \succ A, Y$ $[X, A \succ Y]$ can be transformed into a *procedure* for meeting a justification request for an assertion of A [denial of A] in any available position, appealing only what is granted in $[X : Y]$, and to the defining rules used in that derivation.

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- The *negative* view of the bounds is seen in the clash between assertion and denial, and the *positive* view is found in the answers we can give to justification requests.
- Adopting *defining rules* is one way to be *very* precise about the norms governing the concepts so defined, combining *safety*, *univocity* and *expressive power*.

THANK YOU!

http://consequently.org/presentation/2021/assertion-denial-qa-common-ground-bristol