Assertions, Denials
Questions, Answers
& the Common Ground

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To better understand the speech acts of *assertion* and *denial*, their relationship to *other* speech acts, and connections between these speech acts and logical notions, including the classical sequent calculus.
I want to revisit some themes (and revise some of the claims) in my 2005 paper “Multiple Conclusions.”
The behaviour of two kinds of speech acts: 

*polar (yes/no) questions,*

and *justification requests.*
My Plan

Assertion and Denial

Polar Questions

Positions and Rules

Justification Requests
ASSERTION
AND DENIAL
Multiple Conclusions

\[ X \succ Y \]

Don’t assert each member of \( X \)
and deny each member of \( Y \).
Defining Rules for Logical Concepts

This allows for a uniform, modular system of rules of logical vocabulary.

\[
\begin{align*}
X, A, B & \Rightarrow Y & \land Df \\
X, A \land B & \Rightarrow Y & \\
X, A, B & \Rightarrow Y & \\
X & \Rightarrow A \lor B, Y & \lor Df \\
X & \Rightarrow A \lor B, Y & \\
X & \Rightarrow A, Y & \neg Df \\
X, \neg A & \Rightarrow Y & \\
X & \Rightarrow A \rightarrow B, Y & \rightarrow Df \\
X, A & \Rightarrow B Y & \\
X, A & \Rightarrow B Y & \\
X, A(\mathfrak{n}) & \Rightarrow Y & \forall Df \\
X, \forall x A(x) & \Rightarrow Y & \\
X, A(\mathfrak{n}) & \Rightarrow Y & \exists Df \\
X, \exists x A(x) & \Rightarrow Y & \\
X, F a & \Rightarrow F b, Y & \\
X, F b & \Rightarrow F a, Y & \equiv Df \\
X & \Rightarrow a = b, Y & \\
\end{align*}
\]

Terms & conditions: the singular term \( \mathfrak{n} \) (in \( \forall/\exists Df \)) and the predicate \( F \) (in \( \equiv Df \)) do not appear below the line in those rules.
**Structural Rules**

\[
\begin{align*}
X, A \rightarrow A, Y & \quad Id \\
X \rightarrow A, Y & \quad X, A \rightarrow Y \\
& \quad Cut \\
& \quad X \rightarrow Y
\end{align*}
\]

These rules constrain assertion and denial *as such.*
In appealing to norms governing assertion...

... I was wading into a pre-existing literature about assertion. A *very large* literature.
It is fruitful to think of assertion as an act governed by *norms*. 
Aim to say what is true!
For me: Production Norms

Aim to say what is true!

Only say what you know!
For me: Production Norms

Aim to say what is *true*!

Only say what you *know*!

Be prepared to *back it up* when requested!
The hearer is entitled to re-assert.
The hearer is entitled to re-assert.

You can refer back to the asserter to \textit{vouch for} the assertion.
To assert is to bid for the content asserted to be added to the **COMMON GROUND**, the body of information that we (together) take for granted.
To presuppose something is to take it for granted, or at least to act as if one takes it for granted, as background information as common ground among the participants in the conversation. What is most distinctive about this propositional attitude is that it is a social or public attitude: one presupposes that \( \Phi \) only if one presupposes that others presuppose it as well.

— “Common Ground” *LEP*(2002)
What is the relationship between Assertion and Denial?

In “Multiple Conclusions”, I said little beyond the claim that assertion and denial are incompatible
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What is the relationship between Assertion and Denial?

In “Multiple Conclusions”, I said little beyond the claim that assertion and denial are incompatible (in some sense).

This does not help distinguish denial from retraction, or from other speech acts.
Let’s address this issue...

... by examining polar questions, and their answers, in the light of our background interest in assertion and its norms.
POLAR QUESTIONS
Is it the case that p?

This is a distinct speech act with its own norms.
Is it the case that p?

This is a distinct speech act with its own norms.

It raises an issue.
There are two ways to settle the issue

Yes
There are two ways to settle the issue

Yes  No
The two ways clash

If I say *yes* and you say *no* to some polar question \( p \)?, then we DISAGREE.

That is, we take *different* positions on \( p \).
The two ways clash

If I say *yes* and you say *no*
to some polar question \( p \)?,
then we **DISAGREE**.

That is, we take *different* positions on \( p \).

There is no *shared* position
incorporating both of our answers.
Other responses don’t settle the issue

Other responses, like
Other responses don’t settle the issue

Other responses, like

maybe
Other responses don’t settle the issue

Other responses, like

maybe · I don’t know
Other responses don’t settle the issue

Other responses, like

maybe · I don’t know · I think so
Other responses don’t settle the issue

Other responses, like

*maybe* · *I don’t know* · *I think so*

are acceptable responses to p?,
Other responses don’t settle the issue

Other responses, like

*maybe* · *I don’t know* · *I think so*

are acceptable responses to *p*?,

but they don’t answer the question.

They don’t settle the issue of *p*.
Settling answers are assertions

A *yes* or a *no* to *p?* counts as an assertion.
Settling answers are assertions

A *yes* or a *no* to *p?* counts as an assertion.

(Either answer is governed by all of the assertion norms we’ve seen.)
What does a “no” to $p$ assert?

Presumably $\neg p$. 
What does a “no” to p? assert?

Presumably \( \neg p \).

However, I prefer to think of a *yes* to p? as ruling p *in*, and a *no* to p? as ruling p *out*. 
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However, I prefer to think of a *yes* to $p$ as ruling $p$ in, and a *no* to $p$ as ruling $p$ out.

This way, we can distinguish practices where the *issues* are closed under negation and those with more limited expressive resources.
What does a “no” to \( p \)? assert?

Presumably \( \neg p \).

However, I prefer to think of a \textit{yes} to \( p \)? as ruling \( p \) \textit{in}, and a \textit{no} to \( p \)? as ruling \( p \) \textit{out}.

This way, we can distinguish practices where the \textit{issues} are closed under negation and those with more limited expressive resources.

(Nothing important hangs on this distinction.)
[X : Y]
We have ruled \textit{in} everything in $X$, the \textsc{positive} \textsc{common ground}.
We have ruled *in* everything in X, the *POSITIVE COMMON GROUND*.

We have ruled *out* everything in Y, the *NEGATIVE COMMON GROUND*. 
ABELARD: Astralabe is in the study.
ELOISE: No, he is in the kitchen.
**Denial and Retraction**

**ABELARD**: Astralabe is in the study.

**ELOISE**: No, he is in the kitchen.

**ABELARD**: Astralabe is in the study.

**ELOISE**: No, he is *either* in the kitchen *or* the study.
Denial and Retraction

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**INAPPROPRIATE**
Denial and Retraction

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ELOISE: No, he is in the kitchen.

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ABELARD: Astralabe is in the study.
ELOISE: No, he is *either* in the kitchen *or* the study.

ABELARD: Is Astralabe in the study?
ELOISE: *No, he is *either* in the kitchen *or* the study.

INAPPROPRIATE

ELOISE: Maybe. He’s *either* in the kitchen *or* the study.
Denial and Retraction

ABELARD: Astralabe is in the study.
ELOISE: No, he is in the kitchen.

ABELARD: Is Astralabe in the study?
ELOISE: No, he is in the kitchen.

STRONG DENIAL

ABELARD: Astralabe is in the study.
ELOISE: No, he is *either* in the kitchen *or* the study.

INAPPROPRIATE

ABELARD: Is Astralabe in the study?
ELOISE: *No, he is *either* in the kitchen *or* the study.

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**INAPPROPRIATE**

**ELOISE**: Maybe. He’s *either* in the kitchen *or* the study.

**PARTIAL ANSWER**
Denial and Retraction

ABELARD: Astralabe is in the study.

ELOISE: No, he is in the kitchen.

**STRONG DENIAL**

ABELARD: Astralabe is in the study.

ELOISE: No, he is *either* in the kitchen or the study.

**WEAK DENIAL**

ABELARD: Is Astralabe in the study?

ELOISE: No, he is in the kitchen.

**STRONG DENIAL**

ABELARD: Is Astralabe in the study?

ELOISE: *No, he is *either* in the kitchen or the study.

**INAPPROPRIATE**

ELOISE: Maybe. He’s *either* in the kitchen or the study.

**PARTIAL ANSWER**
Strong and Weak Denial

To strongly deny $p$ is to bid to add $p$ to the negative common ground.
Strong and Weak Denial

To *strongly deny* \( p \) is to bid to add \( p \) to the *negative common ground*.

To *weakly deny* \( p \) is to *block* the addition of \( p \) to the *positive common ground*, or to bid for its *retraction* if it is already there.
Strong or weak denials of \( p \) are appropriate responses to an assertion of \( p \), because the assertion of \( p \) is a bid to add \( p \) to the positive common ground.
Strong and Weak Denial, and the Common Ground

- Strong or weak denials of $p$ are appropriate responses to an assertion of $p$, because the assertion of $p$ is a bid to add $p$ to the positive common ground.

- A strong denial of $p$ is one way to settle the question $p$? — this is generally an appropriate response.
Strong and Weak Denial, and the Common Ground

- Strong or weak denials of \( p \) are appropriate responses to an assertion of \( p \), because the assertion of \( p \) is a bid to add \( p \) to the positive common ground.

- A strong denial of \( p \) is one way to settle the question \( p ? \) — this is generally an appropriate response.

- A weak denial of \( p \) is not generally an appropriate response to the polar question \( p ? \), as the polar question does not place \( p \) in the positive common ground, and the question is inappropriate if \( p \) is already in the positive common ground, so there is no \( p \) to block or retract.
Strong and Weak Denials, and Strong and Weak Assertions

- **STRONG DENIAL**: add to the negative common ground.
Strong and Weak Denials, and Strong and Weak Assertions

- **STRONG DENIAL**: add to the negative common ground.
- **STRONG ASSERTION**: add to the positive common ground.
Strong and Weak Denials, and Strong and Weak Assertions

➤ Strong Denial: add to the negative common ground.
➤ Strong Assertion: add to the positive common ground.
➤ Weak Denial: retract (or block) from the positive common ground.
Strong and Weak Denials, and Strong and Weak Assertions

- **STRONG DENIAL**: add to the negative common ground.
- **STRONG ASSERTION**: add to the positive common ground.
- **WEAK DENIAL**: retract (or block) from the positive common ground.
- **WEAK ASSERTION**: retract (or block) from the negative common ground.
STRONG DENIAL: add to the negative common ground.

STRONG ASSERTION: add to the positive common ground.

WEAK DENIAL: retract (or block) from the positive common ground.

WEAK ASSERTION: retract (or block) from the negative common ground. — “Perhaps p.”
That’s one way to understand the relationship between assertion and denial, and how to distinguish strong denial from other negative speech acts.
One Consequence

The common ground
(what we, together, take for granted)
seems to be very finely grained.
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Abelard is being tutored by
Eloise in geometry. He is
reasoning about a triangle
with interior angles of 40°,
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the angles, and notices that
they sum to 180° …
One Consequence

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*Abelard is being tutored by Eloise in geometry. He is reasoning about a triangle with interior angles of 40°, 60° and 80°. He adds up the angles, and notices that they sum to 180°…*

*ABELARD:* The interior angles of triangles add up to 180°.
One Consequence

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(what we, together, take for granted)
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*Abelard is being tutored by Eloise in geometry. He is reasoning about a triangle with interior angles of 40°, 60° and 80°. He adds up the angles, and notices that they sum to 180°…*

*ABELARD:* The interior angles of triangles add up to 180°.

*ELOISE:* No. The interior angles of *this* triangle add up to 180°. Can you prove the general case?
One Consequence

The common ground
(what we, together, take for granted)
seems to be very finely grained.

*Abelard is being tutored by Eloise in geometry. He is reasoning about a triangle with interior angles of 40°, 60° and 80°. He adds up the angles, and notices that they sum to 180°…*

*ABELARD: The interior angles of triangles add up to 180°.*

*ELOISE: No. The interior angles of this triangle add up to 180°. Can you prove the general case?*

Eloise here seems to block from the common ground (weakly deny) a logical consequence of claims in the common ground (the axioms of geometry), for the same general reason as for other weak denials.
Any position \([X, A : A, Y]\)
in which \(A\) has been
strongly asserted and
strongly denied,
is out of bounds.
Positions

Any position \([X, A : A, Y]\) in which \(A\) has been strongly asserted and strongly denied, is out of bounds.

\[X, A \not\rightarrow A, Y\]
Positions

Any position \([X, A : A, Y]\)
in which \(A\) has been strongly asserted and strongly denied, is out of bounds.

\[
X, A \not\succ A, Y
\]

If \(X \not\equiv Y\) then \([X : Y]\) is *available*.
A Word on Cut

\[
\frac{X \vdash A, Y \quad X, A \vdash Y}{X \vdash Y} \quad \text{Cut}
\]
A Word on Cut

\[ X \supset A, Y \quad X, A \supset Y \]

\[ \frac{\text{Cut}}{X \supset Y} \]

In any available position \([X : Y]\), if one way to settle \(A\) is *not* available, then the other way to settle it *is* available.
POSITIONS
AND RULES
Defining Rules

\[
\begin{align*}
\frac{X, A, B \not\rightarrow Y}{X, A \land B \not\rightarrow Y} & \quad \land Df \\
\frac{X \not\rightarrow A \land B, Y}{X \not\rightarrow A \lor B, Y} & \quad \lor Df \\
\frac{X \not\rightarrow A, Y}{X, \neg A \not\rightarrow Y} & \quad \neg Df \\
\frac{X, A \not\rightarrow B, Y}{X \not\rightarrow A \rightarrow B, Y} & \quad \rightarrow Df
\end{align*}
\]
These are kinds of definitions, showing how to treat assertions or denials of the defined concept in terms of the assertions or denials of their components.
Derivations

\[
\begin{align*}
\neg p & \rightarrow \neg p \\
\therefore & \rightarrow p, \neg p \\
\therefore & \rightarrow p \lor \neg p
\end{align*}
\]

\[-Df \quad \lor Df\]
Derivations

\[
\frac{\neg p \rightarrow \neg p}{\rightarrow p, \neg p} \quad \frac{p \rightarrow p}{p, \neg p \rightarrow} \\
\frac{\rightarrow p \lor \neg p}{\nabla Df} \quad \frac{p \land \neg p \rightarrow}{p \lor \neg p \rightarrow} \\
\frac{p \land \neg p}{\land Df}
\]
Derivations

\[
\begin{align*}
\neg p & \rightarrow \neg p & \neg p & \rightarrow \neg p \\
\rightarrow p, \neg p & \rightarrow p \vee \neg p & \rightarrow p & \rightarrow p \\
\rightarrow p \vee \neg p & \rightarrow p \wedge \neg p & \rightarrow p \wedge \neg p & \rightarrow p \wedge \neg p \\
\neg Df & \neg Df & \neg Df & \neg Df \\
\wedge Df & \wedge Df & \wedge Df & \wedge Df
\end{align*}
\]

\[
\begin{align*}
p, q \vee r & \rightarrow p \wedge q, q \vee r & p \wedge q, q \vee r & \rightarrow p \wedge q, r \\
p, q \vee r & \rightarrow p \wedge q, r \wedge q & q, p, q \vee r & \rightarrow p \wedge q, r \\
p, q \vee r & \rightarrow p \wedge q, r \wedge q & q, p, q \vee r & \rightarrow p \wedge q, r \\
\wedge Df & \wedge Df & \wedge Df & \wedge Df \\
\vee Df & \vee Df & \vee Df & \vee Df \\
\vee Df & \vee Df & \vee Df & \vee Df \\
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\end{align*}
\]
Sequent Derivations aren’t exactly Proofs

They don’t have the same *shape* as proofs.
Sequent Derivations aren’t exactly Proofs

- They don’t have the same \textit{shape} as proofs.
- (Where is the \textit{conclusion} in $p \lor q \Rightarrow p, q$?)
Sequent Derivations aren’t exactly Proofs

➤ They don’t have the same *shape* as proofs.

➤ *(Where is the *conclusion* in $p \lor q \vdash p, q$?)*

➤ A endsequent $X \vdash A$ doesn’t tell you to *infer* $A$ from $X$ — it merely tells you to not assert all members of $X$ and deny $A$. 
“Well, now, let’s take a little bit of the argument in that First Proposition—just two steps, and the conclusion drawn from them. Kindly enter them in your note-book. And in order to refer to them conveniently, let’s call them $A$, $B$, and $Z$:

$A$) Things that are equal to the same are equal to each other.

$B$) The two sides of this Triangle are things that are equal to the same.

$Z$) The two sides of this Triangle are equal to each other.

Readers of Euclid will grant, I suppose, that $Z$ follows logically from $A$ and $B$, so that any one who accepts $A$ and $B$ as true, must accept $Z$ as true?”

“Undoubtedly! The youngest child in a High School—as soon as High Schools are invented, which will not be till some two thousand years later—will grant that.”

“And if some reader had not yet accepted $A$ and $B$ as true, he might still accept the sequence as a valid one, I suppose?”
“Well, now, let’s take a little bit of the argument in that First Proposition—just two steps, and the conclusion drawn from them. Kindly enter them in your note-book. And in order to refer to them conveniently, let’s call them $A$, $B$, and $Z$:–

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Readers of Euclid will grant, I suppose, that $Z$ follows logically from $A$ and $B$, so that any one who accepts $A$ and $B$ as true, must accept $Z$ as true?”

“Undoubtedly! The youngest child in a High School—as soon as High Schools are invented, which will not be till some two thousand years later—will grant that.”

“And if some reader had not yet accepted $A$ and $B$ as true, he might still accept the sequence as a valid one, I suppose?”

The Tortoise never asserts $A$ and $A \rightarrow Z$ while denying $Z$, but he doesn’t accept $A$ and $A \rightarrow Z$ as a reason for $Z$. 
JUSTIFICATION REQUESTS
What is a justification request?

ABELARD: Astralabe is in the kitchen.

ELOISE: Really?

ABELARD: I saw him there five minutes ago.

ELOISE: OK.
What is a justification request?

ABELARD: Astralabe is in the kitchen.
ELOISE: *Really?*
ABELARD: I saw him there five minutes ago.
ELOISE: OK.

ABELARD: Astralabe is in the kitchen.
ELOISE: *Really?*
ABELARD: I saw him there five minutes ago.
ELOISE: *Are you sure?* He’s been in the study with me for the last half hour.
What is a justification request?

ABELARD: Astralabe is in the kitchen.
ELOISE: Really?
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ELOISE: OK.

ABELARD: Astralabe is in the kitchen.
ELOISE: Really?
ABELARD: I saw him there five minutes ago.
ELOISE: Are you sure? He’s been in the study with me for the last half hour.

ABELARD: Astralabe is in the kitchen.
ELOISE: Really?
ABELARD: I saw him there five minutes ago.
ELOISE: Yes, but he was in the study two minutes ago.
We should *expect* the need for justification requests given the commitments and entitlements involved in assertion.
We should *expect* the need for justification requests given the commitments and entitlements involved in assertion.

If I give you permission to ask *me* to vouch for my assertion you should to be able to call me on it.
We should *expect* the need for justification requests given the commitments and entitlements involved in assertion.

If I give you permission to ask *me* to vouch for my assertion you should to be able to call me on it.

That’s a justification request.
What is a justification request?

A justification request for a strong assertion [or strong denial] is an attempt to block the addition to the common ground, until a reason is given.
What is a justification request?

A justification request for a strong assertion [or strong denial] is an attempt to block the addition to the common ground, until a reason is given.

This reason is another assertion [or denial] which must be granted, (added to the common ground) in order for the request to be met.
What is a justification request?

A justification request for a strong assertion [or strong denial] is an attempt to block the addition to the common ground, until a reason is given.

This reason is another assertion [or denial] which must be granted, (added to the common ground) in order for the request to be met.

Granting the given reason is necessary but not sufficient for satisfying the justification request.
Definitions and Justification Requests

ACHILLES So … this is an *equilateral* triangle.

TORTOISE I’m sorry, I don’t follow, my heroic friend. I’ve not heard that word before: what does ‘*equilateral*’ mean?

ACHILLES Oh, that’s easy to explain. ‘*Equilateral*’ means having sides of the same length. An *equilateral* triangle is a triangle with all three sides the same length.

TORTOISE OK. That sounds good. You may continue with your reasoning.

ACHILLES Well, as I was saying, the sides of this triangle are all one cubit in length, so it is an equilateral triangle.

TORTOISE Perhaps you will forgive me, Achilles, but I still don’t follow. I grant to you that the sides of this triangle all have the same length. I fail to see, however, that it *follows* that it is an equilateral triangle. Could you explain why it is?
Definitions and Justification Requests

If I accept the definition $A =_{df} B$, then I should accept granting $A$ as meeting a justification request for the assertion of $B$ and ruling out $A$ as meeting a justification request for $B$’s denial and *vice versa*.

A failure to accept this is a sign that I have not mastered the definition.
What goes for a definition of the form $A =_{df} B$ can also go for defining rules:

$$
\frac{X, A, B \vdash Y}{X, A \land B, \vdash Y} \quad ^{\land Df}
$$
Justification Requests and Defining Rules

What goes for a definition of the form $A =_{df} B$ can also go for defining rules:

$$
X, A, B \vdash Y \\
\frac{}{X, A \land B, \vdash Y} \quad ^{\land Df}
$$

It is a mistake to grant $A$ and grant $B$ and to look for something more to discharge a justification request for an assertion of $A \land B$, if you take $^{\land Df}$ as a definition.
Justification Requests and Defining Rules

\[ \frac{X, A \supset B, Y}{X \supset A \rightarrow B, Y} \rightarrow Df \]
\[
\frac{X, A \not\Rightarrow B, Y}{X \not\Rightarrow A \rightarrow B, Y} \rightarrow Df
\]

It is a mistake to rule A in and rule B out and to look for something more to discharge a justification request for a denial of \( A \rightarrow B \) if you accept \( \rightarrow Df \) as a definition.
A *little* more work is required to show why granting A and A → Z is enough to meet a justification request for Z’s assertion.
A little more work is required to show why granting $A$ and $A \to Z$ is enough to meet a justification request for $Z$’s assertion.

Consider this focused derivation:

\[
\frac{A \to Z > A \to Z}{A \to Z, A > Z} \quad \rightarrow \text{Df}
\]

Read the premise as telling us that in a position in which $A \to Z$ is already ruled in, we have an answer to the justification request for $A \to Z$’s assertion.
A little more work is required to show why granting A and A → Z is enough to meet a justification request for Z’s assertion.

Consider this focussed derivation:

\[
\frac{A \rightarrow Z \quad A \rightarrow Z}{A \rightarrow Z, A \rightarrow Z} \rightarrow \text{Df}
\]

> Read the premise as telling us that in a position in which A → Z is already ruled in, we have an answer to the justification request for A → Z’s assertion.

> Then applying →Df we see why we have an answer to the request concerning Z’s assertion, in a context in which A → Z and A have both been ruled in. (In granting A → Z and A we have settled Z positively. Its denial is ruled out, since to assert A and deny Z amounts to denying A → Z.)
**SLOGAN:** A derivation of $X \rightarrow A$, $Y$ shows us how to meet a justification request for the assertion of $A$ in any available position extending $[X : Y]$. 
**SLOGAN:** A derivation of $X \supset A$, $Y$ shows us how to meet a justification request for the assertion of $A$ in any available position extending $[X : Y]$.

A derivation of $X$, $A \supset Y$ shows us how to meet a justification request for the denial of $A$ in any available position extending $[X : Y]$.
Focussed Structural Rules

\[ X, A \rightarrow A, Y \]

\[ X, A \rightarrow A, Y \]
**Focussed Structural Rules**

\[ X, A \rightarrow A, Y \quad X, A \rightarrow A, Y \]

\[ X, A \rightarrow A, B, Y \quad X, A, B \rightarrow A, Y \]
**Focussed Structural Rules**

\[
\begin{align*}
X, A &\succ A, Y \\
X, A &\succ A, B, Y \\
X &\succ A, Y \\
X, A &\succ B, Y \\
X &\succ B, Y
\end{align*}
\]

\[
\begin{align*}
X, A &\succ A, Y \\
X, A, B &\succ A, Y \\
X, A, B &\succ B, Y \\
X &\succ A, Y \\
X, A, B &\succ Y \\
X, B &\succ Y
\end{align*}
\]
Focussed Structural Rules

X, A \rightarrow \textcolor{red}{A}, Y

X, \textcolor{red}{A} \rightarrow A, Y

X, A \rightarrow A, \textcolor{red}{B}, Y

X, \textcolor{red}{A}, \textcolor{red}{B} \rightarrow A, Y

X \rightarrow \textcolor{red}{A}, Y \hspace{1cm} X, A \rightarrow \textcolor{red}{B}, Y

\text{Cut} \hspace{1cm} \text{Cut}

X \rightarrow \textcolor{red}{B}, Y

X, A, \textcolor{red}{A} \rightarrow Y

X, \textcolor{red}{A} \rightarrow Y

X \rightarrow A, \textcolor{red}{A}, Y

X \rightarrow \textcolor{red}{A}, Y

W \\
W
Swap

\[
\frac{X \vdash A, B, Y \quad X, A \vdash A, B, Y}{X \vdash A, B, B, Y} \quad \text{Cut}
\]

\[
\frac{X \vdash A, B, B, Y}{X \vdash A, B, Y} \quad W
\]
Swap

\[
\begin{align*}
X \supset A, B, Y \quad & \quad X, A \supset A, B, Y \\
& \quad \frac{\quad}{\quad X \supset A, B, B, Y} \quad W \\
& \quad \frac{\quad}{\quad X \supset A, B, Y} \quad \text{Cut}
\end{align*}
\]

\[
\begin{align*}
X \supset A, B, Y \\
& \quad \frac{\quad}{\quad A, B, Y} \quad \text{Swap}
\end{align*}
\]
Focussed Defining Rules

\[
\begin{align*}
X, A, B & > Y \quad \land Df \\
X, A \land B & > Y
\end{align*}
\]

\[
\begin{align*}
X, A, B & > Y \quad \land Df \\
X, A \land B & > Y
\end{align*}
\]

\[
\begin{align*}
X > A, B, Y \quad \lor Df \\
X > A \lor B, Y
\end{align*}
\]

\[
\begin{align*}
X > A, B, Y \quad \lor Df \\
X > A \lor B, Y
\end{align*}
\]

\[
\begin{align*}
X > A, Y \quad \neg Df \\
X > A, \neg Y
\end{align*}
\]

\[
\begin{align*}
X > A, B, Y \quad \rightarrow Df \\
X > A \rightarrow B, Y
\end{align*}
\]

\[
\begin{align*}
X > A, B, Y \quad \rightarrow Df \\
X > A \rightarrow B, Y
\end{align*}
\]
To prove $A \to B$, *rule A in* (suppose it) and prove $B$.

Or, *rule B out* (suppose it), and refute $A$.
This can be represented as a *dialogue*, meeting a justification request for an assertion of \((p \rightarrow q) \rightarrow p\) → p. (See the handout for an example.)
Now that we see how focussed derivations can be seen as procedures for meeting justification requests, we have answers to our original questions and concerns about the sequent calculus.
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- If we understand a *conclusion* of a *proof* the meeting of a justification request, we can see why this kind of conclusion is *single*.
Answers!

Now that we see how focussed derivations can be seen as procedures for meeting justification requests, we have answers to our original questions and concerns about the sequent calculus.

- If we understand a conclusion of a proof the meeting of a justification request, we can see why this kind of conclusion is single.

- Since both assertions and denials can be the target of a justification request, this single conclusion can be in the right or the left of a sequent.
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- The making of an inference is a (possibly preemptive) answer to a justification request.

- A derivation of a sequent $X \rightarrow A, Y [X, A \rightarrow Y]$ can be transformed into a procedure for meeting a justification request for an assertion of $A$ [denial of $A$] in any available position, appealing only what is granted in $[X: Y]$, and to the defining rules used in that derivation.
The value of derivations

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- Derivations are one way we can grasp complex bounds and enforce them.

- The negative view of the bounds is seen in the clash between assertion and denial, and the positive view is found in the answers we can give to justification requests.

- Adopting defining rules is one way to be very precise about the norms governing the concepts so defined, combining safety, univocity and expressive power.
THANK YOU!