My Plan

Background

A Definition

A Method ...

... and its Cost

Preservation

Examples

The Upshot
BACKGROUND
I’m interested in disagreement...
I’m interested in *disagreement*...

...and I’m interested in *words*, and what they mean.
In particular, I’m interested in the role that logic and logical concepts might play in clarifying and managing disagreement.
Particular Issues

- Disagreement between rival accounts of logic
Particular Issues

- Disagreement between rival accounts of logic
- Monism and Pluralism about logic
• Disagreement between rival accounts of logic
• Monism and Pluralism about logic
• Ontological relativity (_EXISTS)
Particular Issues

- Disagreement between rival accounts of logic
- Monism and Pluralism about logic
- Ontological relativity ($\Xi$)
- The status of modal vocabulary ($\Diamond$)
There's a lady who's sure all that glitters is gold
And she's buying a stairway to heaven.
When she gets there she knows, if the stores are all closed
With a word she can get what she came for.
Ooh, ooh, and she's buying a stairway to heaven.

There's a sign on the wall but she wants to be sure
'Cause you know sometimes words have two meanings.
In a tree by the brook, there's a songbird who sings,
Sometimes all of our thoughts are misgiven.

Ooh, it makes me wonder, Ooh, it makes me wonder.

There's a feeling I get when I look to the west,
And my spirit is crying for leaving.
In my thoughts I have seen rings of smoke through the trees,
And the voices of those who stand looking.

Ooh, it makes me wonder, Ooh, it really makes me wonder.

And it's whispered that soon, if we all call the tune,
Then the piper will lead us to reason.
And a new day will dawn for those who stand long,
And the forests will echo with laughter.

If there's a bustle in your hedgerow, don't be alarmed now,
It's just a spring clean for the May Queen.
Yes, there are two paths you can go by, but in the long run
There's still time to change the road you're on.

And it makes me wonder.

Your head is humming and it won't go, in case you don't know,
The piper's calling you to join him,
Dear lady, can you hear the wind blow, and did you know
Your stairway lies on the whispering wind?

And as we wind on down the road
Our shadows taller than our soul.
There walks a lady we all know
Who shines white light and wants to show
How everything still turns to gold.
And if you listen very hard
The tune will come to you at last.
When all are one and one is all
To be a rock and not to roll.

And she's buying a stairway to heaven.
There’s a lady who’s sure all that glitters is gold
And she’s buying a stairway to heaven.
When she gets there she knows, if the stores are all closed
With a word she can get what she came for.
Ooh, ooh, and she’s buying a stairway to heaven.

There’s a sign on the wall but she wants to be sure
’Cause you know sometimes words have two meanings.
In a tree by the brook, there’s a songbird who sings,
Sometimes all of our thoughts are misgiven.

Ooh, it makes me wonder, Ooh, it makes me wonder.

There’s a feeling I get when I look to the west,
And my spirit is crying for leaving.
In my thoughts I have seen rings of smoke through the trees,
And the voices of those who stand looking.

Ooh, it makes me wonder, Ooh, it really makes me wonder.

And it’s whispered that soon, if we all call the tune,
Then the piper will lead us to reason.
And a new day will dawn for those who stand long,
And the forests will echo with laughter.

If there’s a bustle in your hedgerow, don’t be alarmed now,
It’s just a spring clean for the May Queen.
Yes, there are two paths you can go by, but in the long run
There’s still time to change the road you’re on.

And it makes me wonder.

Your head is humming and it won’t go, in case you don’t know,
The piper’s calling you to join him,
Dear lady, can you hear the wind blow, and did you know
Your stairway lies on the whispering wind?

And as we wind on down the road
Our shadows taller than our soul.
There walks a lady we all know
Who shines white light and wants to show
How everything still turns to gold.
And if you listen very hard
The tune will come to you at last.
When all are one and one is all
To be a rock and not to roll.

And she’s buying a stairway to heaven.
Let Led Zeppelin Explain...

There's a lady who's sure all that glitters is gold
And she's buying a stairway to heaven.
When she gets there she knows, if the stores are all closed
With a word she can get what she came for.
Ooh, ooh, and she's buying a stairway to heaven.

There's a sign on the wall but she wants to be sure
'Cause you know sometimes words have two meanings.
In a tree by the brook, there's a songbird who sings,
Sometimes all of our thoughts are misgiven.

Ooh, it makes me wonder, Ooh, it makes me wonder.

There's a feeling I get when I look to the west,
And my spirit is crying for leaving.
In my thoughts I have seen rings of smoke through the trees,
And the voices of those who stand looking.

Ooh, it makes me wonder, Ooh, it really makes me wonder.

And it's whispered that soon, if we all call the tune,
Then the piper will lead us to reason.
And a new day will dawn for those who stand long,
And the forests will echo with laughter.

If there's a bustle in your hedgerow, don't be alarmed now,
It's just a spring clean for the May Queen.
Yes, there are two paths you can go by, but in the long run
There's still time to change the road you're on.

And it makes me wonder.

Your head is humming and it won't go, in case you don't know,
The piper's calling you to join him,
Dear lady, can you hear the wind blow, and did you know
Your stairway lies on the whispering wind?

And as we wind on down the road
Our shadows taller than our soul.
There walks a lady we all know
Who shines white light and wants to show
How everything still turns to gold.
And if you listen very hard
The tune will come to you at last.
When all are one and one is all
To be a rock and not to roll.

And she's buying a stairway to heaven.
There's a lady who's sure all that glitters is gold
And she's buying a stairway to heaven.
When she gets there she knows, if the stores are all closed
With a word she can get what she came for.
Ooh, ooh, and she's buying a stairway to heaven.

There's a sign on the wall but she wants to be sure
'Cause you know sometimes words have two meanings.
In a tree by the brook, there's a songbird who sings,
Sometimes all of our thoughts are misgiven.

Ooh, it makes me wonder, Ooh, it makes me wonder.

There's a feeling I get when I look to the west,
And my spirit is crying for leaving.
In my thoughts I have seen rings of smoke through the trees,
And the voices of those who stand looking.

Ooh, it makes me wonder, Ooh, it really makes me wonder.

And it's whispered that soon, if we all call the tune,
Then the piper will lead us to reason.
And a new day will dawn for those who stand long,
And the forests will echo with laughter.

If there's a bustle in your hedgerow, don't be alarmed now,
It's just a spring clean for the May Queen.
Yes, there are two paths you can go by, but in the long run
There's still time to change the road you're on.

And it makes me wonder.

Your head is humming and it won't go, in case you don't know,
The piper's calling you to join him,
Dear lady, can you hear the wind blow, and did you know
Your stairway lies on the whispering wind?

And as we wind on down the road
Our shadows taller than our soul.
There walks a lady we all know
Who shines white light and wants to show
How everything still turns to gold.
And if you listen very hard
The tune will come to you at last.
When all are one and one is all
To be a rock and not to roll.

And she's buying a stairway to heaven.
Let Led Zeppelin Explain...

There's a lady who's sure all that glitters is gold
And she's buying a stairway to heaven.
When she gets there she knows, if the stores are all closed
With a word she can get what she came for.
Ooh, ooh, and she's buying a stairway to heaven.

If there's a bustle in your hedgerow, don't be alarmed now,
It's just a spring clean for the May Queen.
Yes, there are two paths you can go by, but in the long run
There's still time to change the road you're on.

And it makes me wonder.

Your head is humming and it won't go, in case you don't know,
The piper's calling you to join him,
Dear lady, can you hear the wind blow, and did you know
Your stairway lies on the whispering wind?

And as we wind on down the road
Our shadows taller than our soul.
There walks a lady we all know
Who shines white light and wants to show
How everything still turns to gold.
And if you listen very hard
The tune will come to you at last.
When all are one and one is all
To be a rock and not to roll.

And she's buying a stairway to heaven.

**conjunction 19  negation 3  existential quantifier 15  possibility 2**
There's a lady who's sure all that glitters is gold
And she's buying a stairway to heaven.
When she gets there she knows, if the stores are all closed
With a word she can get what she came for.
Ooh, ooh, and she's buying a stairway to heaven.

If there's a bustle in your hedgerow, don't be alarmed now,
It's just a spring clean for the May Queen.
Yes, there are two paths you can go by, but in the long run
There's still time to change the road you're on.

And it makes me wonder.
Your head is humming and it won't go, in case you don't know,
The piper's calling you to join him,
Dear lady, can you hear the wind blow, and did you know
Your stairway lies on the whispering wind?

And as we wind on down the road
Our shadows taller than our soul.
There walks a lady we all know
Who shines white light and wants to show
How everything still turns to gold.
And if you listen very hard
The tune will come to you at last.
When all are one and one is all
To be a rock and not to roll.

And she's buying a stairway to heaven.
Let Led Zeppelin Explain...

There's a lady who's sure all that glitters is gold
And she's buying a stairway to heaven.
When she gets there she knows, if the stores are all closed
With a word she can get what she came for.
Ooh, ooh, and she's buying a stairway to heaven.

There's a sign on the wall but she wants to be sure
'Cause you know sometimes words have two meanings.
In a tree by the brook, there's a songbird who sings,
Sometimes all of our thoughts are misgiven.

Ooh, it makes me wonder, Ooh, it makes me wonder.

There's a feeling I get when I look to the west,
And my spirit is crying for leaving.
In my thoughts I have seen rings of smoke through the trees,
And the voices of those who stand looking.

Ooh, it makes me wonder, Ooh, it really makes me wonder.

And it's whispered that soon, if we all call the tune,
Then the piper will lead us to reason.
And a new day will dawn for those who stand long,
And the forests will echo with laughter.

If there's a bustle in your hedgerow, don't be alarmed now,
It's just a spring clean for the May Queen.
Yes, there are two paths you can go by, but in the long run
There's still time to change the road you're on.

And it makes me wonder.

Your head is humming and it won't go, in case you don't know,
The piper's calling you to join him,
Dear lady, can you hear the wind blow, and did you know
Your stairway lies on the whispering wind?

And as we wind on down the road
Our shadows taller than our soul.
There walks a lady we all know
Who shines white light and wants to show
How everything still turns to gold.
And if you listen very hard
The tune will come to you at last.
When all are one and one is all
To be a rock and not to roll.

And she's buying a stairway to heaven.

conjunction 19  negation 3  existential quantifier 15  possibility 2
A DEFINITION
A man walks rapidly around a tree, while a squirrel moves on the tree trunk. Both face the tree at all times, but the tree trunk stays between them. A group of people are arguing over the question:
A man walks rapidly around a tree, while a squirrel moves on the tree trunk. Both face the tree at all times, but the tree trunk stays between them. A group of people are arguing over the question:

Does the man go round the squirrel or not?
A man walks rapidly around a tree, while a squirrel moves on the tree trunk. Both face the tree at all times, but the tree trunk stays between them. A group of people are arguing over the question:

Does the man go round the squirrel or not?

α: The man goes round the squirrel.

δ: The man doesn’t go round the squirrel.
Which party is right depends on what you practically mean by ‘going round’ the squirrel. If you mean passing from the north of him to the east, then to the south, then to the west, and then to the north of him again, obviously the man does go round him, for he occupies these successive positions. But if on the contrary you mean being first in front of him, then on the right of him then behind him, then on his left, and finally in front again, it is quite as obvious that the man fails to go round him ...

Make the distinction, and there is no occasion for any farther dispute.

— William James, Pragmatism (1907)
Resolving a dispute by clarifying meanings

$\alpha$: The man \textit{goes round}_1 the squirrel.

$\delta$: The man doesn’t \textit{go round}_2 the squirrel.
Resolving a dispute by clarifying meanings

α: The man goes round\(_1\) the squirrel.

δ: The man doesn’t go round\(_2\) the squirrel.

Once we disambiguate “going round” no disagreement remains.
For James, “going round$_1$” and “going round$_2$” are explicated in other terms of $\alpha$ and $\delta$’s vocabulary.
For James, “going round$_1$” and “going round$_2$” are explicated in other terms of $\alpha$ and $\delta$’s vocabulary.

Perhaps terms $t_1$ and $t_2$ can’t be explicated in terms of prior vocabulary. No matter.
For James, “going round$_1$” and “going round$_2$” are explicated in other terms of $\alpha$ and $\delta$’s vocabulary.

Perhaps terms $t_1$ and $t_2$ can’t be explicated in terms of prior vocabulary. No matter.

$\alpha$ could learn $t_2$ while $\delta$ could learn $t_1$. 
Introducing General Scheme

\[ L_\alpha \quad A \quad L_\delta \]
Introducing General Scheme

\[ L_\alpha \rightarrow t_\alpha(A) \rightarrow t_\delta(A) \rightarrow L_* \rightarrow L_\delta \]
What is a *Language*

▶ A syntax ▶ positions \[ X:Y \], where each member of \( X \) is asserted and each member of \( Y \) is denied, which are either ▶ incoherent (out of bounds) \( X \vdash Y \), or ▶ coherent (in bounds) \( X \not\vdash Y \).

▶ identity: \( A \vdash A \).

▶ weakening: If \( X \vdash Y \) then \( X; A \vdash Y \) and \( X \vdash A; Y \).

▶ cut: If \( X \vdash A; Y \) and \( X; A \vdash Y \) then \( X \vdash Y \).
What is a Language?

- **A Syntax**

  ▶ A syntax positions $[X: Y]$, where each member of $X$ is asserted and each member of $Y$ is denied, which are either incoherent (out of bounds) $X \vdash Y$, or coherent (in bounds) $X \not\vdash Y$.

+ identity: $A \vdash A$.

+ weakening: If $X \vdash Y$ then $X; A \vdash Y$ and $X \vdash A; Y$.

+ cut: If $X \vdash A; Y$ and $X; A \vdash Y$ then $X \vdash Y$. 

Greg Restall
http://consequently.org/presentation/2015/verbal-disputes-oxford/
What is a *Language*?

- **A Syntax**
- **Positions** \([X : Y]\), where each member of \(X\) is *asserted* and each member of \(Y\) is *denied*,

Greg Restall

http://consequently.org/presentation/2015/verbal-disputes-oxford/
What is a Language?

- **A Syntax**
- **Positions** \([X : Y]\), where each member of \(X\) is *asserted* and each member of \(Y\) is *denied*,

which are either **incoherent** (out of bounds) \(X \vdash Y\),
A syntax

POSITIONS \([X : Y]\), where each member of \(X\) is asserted and each member of \(Y\) is denied,

which are either INCOHERENT (out of bounds) \(X \vdash Y\), or COHERENT (in bounds) \(X \nvdash Y\).
What is a Language?

- **A Syntax**
- **Positions** \([X : Y]\), where each member of \(X\) is *asserted* and each member of \(Y\) is *denied*,

which are either **incoherent** *(out of bounds)* \(X \vdash Y\),

or **coherent** *(in bounds)* \(X \nvdash Y\).

+ **Identity**: \(A \vdash A\).
What is a Language?

- **A SYNTAX**

- **POSITIONS** \([X : Y]\), where each member of \(X\) is *asserted* and each member of \(Y\) is *denied*,

  which are either **INCOHERENT** (*out of bounds*) \(X \vdash Y\),
  or **COHERENT** (*in bounds*) \(X \nvdash Y\).

  + **IDENTITY**: \(A \vdash A\).

  + **WEAKENING**: If \(X \vdash Y\) then \(X, A \vdash Y\) and \(X \vdash A, Y\).
What is a Language?

- **A syntax**
- **positions** \([X : Y]\), where each member of \(X\) is asserted and each member of \(Y\) is denied,

which are either **incoherent** (out of bounds) \(X \vdash Y\), or **coherent** (in bounds) \(X \nvdash Y\).

+ **Identity**: \(A \vdash A\).
+ **Weakening**: If \(X \vdash Y\) then \(X, A \vdash Y\) and \(X \vdash A, Y\).
+ **Cut**: If \(X \vdash A, Y\) and \(X, A \vdash Y\) then \(X \vdash Y\).
What is a Translation?

- Incoherence preserving:
  \[ X \vdash L_1 \rightarrow Y \]
  \[ X \vdash L_2 \rightarrow Y \]

- Coherence preserving:
  \[ X \nvdash L_1 \rightarrow Y \]
  \[ X \nvdash L_2 \rightarrow Y \]

- Compositional (e.g.,
  \[ t(A \land B) = (t(A) \land t(B)) \]
  \[ t(p : q : (p \land q)) = p : q : (p \land q) \]

Greg Restall
http://consequently.org/presentation/2015/verbal-disputes-oxford/
What is a Translation?

\[ t : L_1 \rightarrow L_2 \]
What is a Translation?

\[ t : L_1 \rightarrow L_2 \]

- \( t \) may be **INCOHERENCE PRESERVING**: \( X \vdash_{L_1} Y \Rightarrow t(X) \vdash_{L_2} t(Y) \).
What is a Translation?

\[ t : L_1 \rightarrow L_2 \]

- t may be **INCOHERENCE PRESERVING**: \( X \vdash_{L_1} Y \Rightarrow t(X) \vdash_{L_2} t(Y) \).
- t may be **COHERENCE PRESERVING**: \( X \not\vdash_{L_1} Y \Rightarrow t(X) \not\vdash_{L_2} t(Y) \).
What is a Translation?

\[ t : L_1 \rightarrow L_2 \]

- \( t \) may be **INCOHERENCE PRESERVING**: \( X \vdash_{L_1} Y \Rightarrow t(X) \vdash_{L_2} t(Y) \).
- \( t \) may be **COHERENCE PRESERVING**: \( X \not\vdash_{L_1} Y \Rightarrow t(X) \not\vdash_{L_2} t(Y) \).
- \( t \) may be **COMPOSITIONAL** (e.g., \( t(A \land B) = \neg(\neg t(A) \lor \neg t(A)) \)), so \( t(\lambda p.\lambda q.(p \land q)) = \lambda p.\lambda q.(\neg(\neg p \lor \neg q)) \).
Example Translations

- $t_\alpha(\text{going round}) = \text{going round}_1$; $t_\delta(\text{going round}) = \text{going round}_2$. 
Example Translations

- $t_\alpha(\text{going round}) = \text{going round}_1$; $t_\delta(\text{going round}) = \text{going round}_2$.

- $\text{dm} : L[\land, \lor, \neg] \rightarrow L[\lor, \neg]$, a de Morgan translation.  
$\text{dm}(A \land B) = \neg(\neg\text{dm}(A) \lor \neg\text{dm}(B))$. This is coherence and incoherence preserving, and compositional.
Example Translations

- \( t_\alpha(\text{going round}) = \text{going round}_1; \ t_\delta(\text{going round}) = \text{going round}_2. \)
- \( \text{dm} : L[\land, \lor, \neg] \rightarrow L[\lor, \neg], \) a \textit{de Morgan} translation. \( \text{dm}(A \land B) = \neg(\neg\text{dm}(A) \lor \neg\text{dm}(B)). \) This is \textit{coherence} and \textit{incoherence preserving}, and \textit{compositional}.
- \( s : L[0, ', +, \times] \rightarrow L[\in], \) interpreting arithmetic into set theory.
Example Translations

- $t_{\alpha}(\text{going round}) = \text{going round}_1; t_{\delta}(\text{going round}) = \text{going round}_2.$

- $dm : L[\land, \lor, \neg] \rightarrow L[\lor, \neg]$, a de Morgan translation. 
  $dm(A \land B) = \neg(\neg dm(A) \lor \neg dm(B)).$ This is coherence and incoherence preserving, and compositional.

- $s : L[0, ', +, \times] \rightarrow L[\in],$ interpreting arithmetic into set theory.

  This is compositional and coherence preserving, but not incoherence preserving for FOL derivability. $(\forall x)(\exists y)(y = x + 1)$ is true in all models (whether the axioms of PA hold or not). Its translation $(\forall x \in \omega)(\exists y \in \omega)(\forall z)(z \in y \equiv (z \in x \lor z = x))$ is a ZF theorem but not true in all models.
Example Translations

- \( t_\alpha(\text{going round}) = \text{going round}_1; \ t_\delta(\text{going round}) = \text{going round}_2. \)

- \( \text{dm} : L[\land, \lor, \neg] \to L[\lor, \neg] \), a de Morgan translation. 
  \( \text{dm}(A \land B) = \neg(\neg\text{dm}(A) \lor \neg\text{dm}(B)) \). This is coherence and incoherence preserving, and compositional.

- \( s : L[0, ', +, \times] \to L[\in], \) interpreting arithmetic into set theory. 
  This is compositional and coherence preserving, but not incoherence preserving for FOL derivability. \( (\forall x)(\exists y)(y = x + 1) \) is true in all models (whether the axioms of PA hold or not). Its translation \( (\forall x \in \omega)(\exists y \in \omega)(\forall z)(z \in y \equiv (z \in x \lor z = x)) \) is a ZF theorem but not true in all models.
  \( \vdash (\forall x)(\exists y)(y = x + 1) \) while \( \not\vdash t[(\forall x)(\exists y)(y = x + 1)] \).
A dispute...
A dispute between a speaker $\alpha$ of language $L_\alpha$. 
A dispute between a speaker $\alpha$ of language $L_\alpha$, and $\delta$ of language $L_\delta$, 
A dispute between a speaker $\alpha$ of language $L_\alpha$, and $\delta$ of language $L_\delta$, over $C$
A dispute between a speaker $\alpha$ of language $L_\alpha$, and $\delta$ of language $L_\delta$, over $C$ (where $\alpha$ asserts $C$ and $\delta$ denies $C$)
A dispute between a speaker $\alpha$ of language $L_{\alpha}$, and $\delta$ of language $L_{\delta}$, over $C$ (where $\alpha$ asserts $C$ and $\delta$ denies $C$) is said to be **RESOLVED BY TRANSLATIONS** $t_{\alpha}$ AND $t_{\delta}$ iff
A dispute between a speaker $\alpha$ of language $L_\alpha$, and $\delta$ of language $L_\delta$, over $C$ (where $\alpha$ asserts $C$ and $\delta$ denies $C$) is said to be **RESOLVED BY TRANSLATIONS** $t_\alpha$ AND $t_\delta$ iff

- For some language $L_*$, $t_\alpha : L_\alpha \rightarrow L_*$, and $t_\delta : L_\delta \rightarrow L_*$,
A dispute between a speaker $\alpha$ of language $L_\alpha$, and $\delta$ of language $L_\delta$, over $C$ (where $\alpha$ asserts $C$ and $\delta$ denies $C$) is said to be **RESOLVED BY TRANSLATIONS $t_\alpha$ AND $t_\delta$ iff**

- For some language $L_*$, $t_\alpha : L_\alpha \rightarrow L_*$, and $t_\delta : L_\delta \rightarrow L_*$,
- and $t_\alpha(C) \not\vdash_{L_*} t_\delta(C)$. 

[Greg Restall](http://consequently.org/presentation/2015/verbal-disputes-oxford/)
Given a resolution by translation, there is no disagreement over $C$ in the shared language $L$. 
Given a resolution by translation, there is no disagreement over $C$ in the shared language $L_*$. The position $[t_\alpha(C) : t_\delta(C)]$ (in $L_*$) is coherent.
To *take* a dispute to be resolved by translation is to take there to be a pair of translations that resolves the dispute.
To *take* a dispute to be resolved by translation is to take there to be a pair of translations that resolves the dispute.

(You may not even *have* the translations in hand.)
A METHOD ...
... to resolve *any* dispute by translation.
Resolution by Disjoint Union
Or, what I like to call “the way of the undergraduate relativist.”
Resolution by Disjoint Union

\[ L_{\alpha|\delta} = L_{\alpha} \sqcup L_{\delta} \]
Resolution by Disjoint Union

\[ L_\alpha | \delta = L_\alpha \sqcup L_\delta \]
$L_{\alpha|\delta}$ is the *disjoint union* $L_\alpha \sqcup L_\delta$, and $t_\alpha : L_\alpha \rightarrow L_{\alpha|\delta}, t_\delta : L_\delta \rightarrow L_{\alpha|\delta}$ are the obvious injections.

For coherence on $L_{\alpha|\delta}$, $(X;X \parallel Y;Y)$ iff $(X \parallel Y)$ or $(X \parallel Y)$. This is a coherence relation. The vocabularies slide past one another with no interaction. This ‘translation’ is structure-preserving, and coherence and incoherence preserving too.
Resolution by Disjoint Union

$L_{\alpha|\delta}$ is the disjoint union $L_{\alpha} \sqcup L_{\delta}$, and $t_{\alpha} : L_{\alpha} \rightarrow L_{\alpha|\delta}$, $t_{\delta} : L_{\delta} \rightarrow L_{\alpha|\delta}$ are the obvious injections.

For coherence on $L_{\alpha|\delta}$,

$$(X_{\alpha}, X_{\delta} \vdash Y_{\alpha}, Y_{\delta}) \text{ iff } (X_{\alpha} \vdash Y_{\alpha}) \text{ or } (X_{\delta} \vdash Y_{\delta}).$$
Resolution by Disjoint Union

$L_{\alpha|\delta}$ is the disjoint union $L_\alpha \sqcup L_\delta$, and $t_\alpha : L_\alpha \rightarrow L_{\alpha|\delta}$, $t_\delta : L_\delta \rightarrow L_{\alpha|\delta}$ are the obvious injections.

For coherence on $L_{\alpha|\delta}$, $(X_\alpha, X_\delta \vdash Y_\alpha, Y_\delta)$ iff $(X_\alpha \vdash Y_\alpha)$ or $(X_\delta \vdash Y_\delta)$.

This is a coherence relation. The vocabularies *slide past one another* with no interaction.
Resolution by Disjoint Union

\[ L_{\alpha|\delta} \text{ is the disjoint union } L_\alpha \sqcup L_\delta, \]
and \( t_\alpha : L_\alpha \rightarrow L_{\alpha|\delta}, \ t_\delta : L_\delta \rightarrow L_{\alpha|\delta} \)
are the obvious injections.

For coherence on \( L_{\alpha|\delta} \),
\[(X_\alpha, X_\delta \vdash Y_\alpha, Y_\delta) \text{ iff } (X_\alpha \vdash Y_\alpha) \text{ or } (X_\delta \vdash Y_\delta).\]

This is a coherence relation.
The vocabularies slide past one another with no interaction.

This ‘translation’ is structure preserving, and coherence and incoherence preserving too.
This ‘resolves’ the dispute over C

If $C \not\models_{L_{\alpha}}$
This ‘resolves’ the dispute over C

If $C \not\vdash_{\alpha}$

($\alpha$’s assertion of $C$ is coherent)
This ‘resolves’ the dispute over C

If $C \not\vdash_{L\alpha} \not\vdash_{L\delta} C$

(α’s assertion of $C$ is coherent)

and $\not\vdash_{L\delta} C$
This ‘resolves’ the dispute over $C$

If $C \vdash_{L_\alpha}$

($\alpha$’s assertion of $C$ is coherent)

and $\not\vdash_{L_\delta} C$

($\delta$’s denial of $C$ is coherent)
This ‘resolves’ the dispute over C

If $C \vdash_{L_\alpha}$

($\alpha$’s assertion of $C$ is coherent)

and $\vdash_{L_\delta} C$

($\delta$’s denial of $C$ is coherent)

then $C \vdash_{L_{\alpha|\delta}} C$
This ‘resolves’ the dispute over $C$

If $C \nvdash_{L_\alpha} (\alpha$’s assertion of $C$ is coherent)

and $\vdash_{L_\delta} C (\delta$’s denial of $C$ is coherent)

then $C \nvdash_{L_{\alpha|\delta}} C (Asserting $C$-from-$L_\alpha$ and denying $C$-from-$L_\delta$ is coherent.)
... AND ITS COST
Nothing $\alpha$ says has any bearing on $\delta$, or vice versa.
What is $A \land B$?
What is $A \land B$?

There’s no such sentence in $L_{\alpha|\delta}$!
Suppose aliens land on earth speaking our languages and familiar with our cultures and tell us that for more complete communication it will be necessary that we increase our vocabulary by the addition of a 1-ary sentence connective \( \forall \) ... concerning which we should note immediately that certain restrictions to our familiar inferential practices will need to be imposed. As these Venusian logicians explain, \( (\land E) \) will have to be curtailed. Although for purely terrestrial sentences \( A \) and \( B \), each of \( A \) and \( B \) follows from their conjunction \( A \land B \), it will not in general be the case that \( \forall A \) follows from \( \forall A \land B \), or that \( \forall B \) follows from \( A \land \forall B \)...

— Lloyd Humberstone, *The Connectives* §4.34
If some statements $A$ (from $L_\alpha$) and $B$ (from $L_\delta$) are both deniable (so $\not\vdash A$, and $\not\vdash B$) then no sentence in $L_{\alpha|\delta}$ entails both $A$ and $B$. 
If some statements $A$ (from $L_\alpha$) and $B$ (from $L_\delta$) are both *deniable* (so $\not\vdash A$, and $\not\vdash B$) then no sentence in $L_{\alpha|\delta}$ entails both $A$ and $B$.

If $C \vdash A$ and $C \vdash B$ then
Losing our Conjunction

If some statements $A$ (from $L_\alpha$) and $B$ (from $L_\delta$) are both deniable (so $\not\vdash A$, and $\not\vdash B$) then no sentence in $L_{\alpha|\delta}$ entails both $A$ and $B$.

If $C \vdash A$ and $C \vdash B$ then

- if $C$ is in $L_\alpha$ then $C \vdash A$ (possible) and $\vdash B$ (no).
If some statements $A$ (from $L_\alpha$) and $B$ (from $L_\delta$) are both deniable (so $\nvdash A$, and $\nvdash B$) then no sentence in $L_{\alpha|\delta}$ entails both $A$ and $B$.

If $C \vdash A$ and $C \vdash B$ then

- if $C$ is in $L_\alpha$ then $C \vdash A$ (possible) and $\vdash B$ (no).
- if $C$ is in $L_\delta$ then $C \vdash B$ (possible) and $\vdash C$ (no).
Losing our Conjunction

If some statements $A$ (from $L_{\alpha}$) and $B$ (from $L_{\delta}$) are both deniable (so $\not\models A$, and $\not\models B$) then no sentence in $L_{\alpha|\delta}$ entails both $A$ and $B$.

If $C \models A$ and $C \models B$ then

- if $C$ is in $L_{\alpha}$ then $C \models A$ (possible) and $\not\models B$ (no).
- if $C$ is in $L_{\delta}$ then $C \models B$ (possible) and $\not\models C$ (no).

So, there’s no conjunction in $L_{\alpha|\delta}$. 
PRESERVATION
Have we got conjunction in $L$?

We can mean many different things by 'and'. Let's say that 'and' is a conjunction in $L$ iff:

$$X; A; B \vdash Y \iff \downarrow \downarrow X; A \text{ and } B \vdash Y$$

for all $X$, $Y$, $A$ and $B$ in $L$.

(There is no conjunction in $L_j$. There is no sentence "A and B".)
Have we got conjunction in $L$?

We can mean *many* different things by ‘and’.
Have we got conjunction in $L$?

We can mean *many* different things by ‘and’.

Let’s say that ‘and’ is a *conjunction* in $L$ iff:

$\exists X; A; B 
\forall X; Y; A; B 
\vdash Y = \text{and} \downarrow X; A \text{ and } B \vdash Y$
Have we got conjunction in \( L \)?

We can mean *many* different things by ‘and’.

Let’s say that ‘and’ is a *conjunction* in \( L \) iff:

\[
\frac{X, A, B \vdash Y}{X, A \text{ and } B \vdash Y} \quad \text{[and\( \uparrow \)]}
\]

for all \( X, Y, A \) and \( B \) in \( L \).
Have we got conjunction in $L$?

We can mean *many* different things by ‘and’.

Let’s say that ‘and’ is a *conjunction* in $L$ iff:

$$
\begin{align*}
X, A, B & \vdash Y \\
\frac{X, A, B \vdash Y}{X, A \text{ and } B \vdash Y} \quad \text{[and]} \\
\end{align*}
$$

for all $X, Y, A$ and $B$ in $L$.

(There is no conjunction in $L_{\alpha|\delta}$. There is no sentence “$A$ and $B$”.)
A translation $t : L_1 \rightarrow L_2$ is **CONJUNCTION PRESERVING** if a conjunction in $L_1$ is translated by a conjunction in $L_2$. 
Preservation seems like a good idea

Translations should keep *some things* preserved.

Let’s see what we can do with this.
Obviously, there some disagreements can resolved by a disambiguation of different senses of the word ‘and.’
Conjunction

Obviously, there are some disagreements that can be resolved by a disambiguation of different senses of the word ‘and.’

\[ \text{‘and}_\alpha \xrightarrow{t_\alpha} \wedge \quad \text{‘and}_\delta \xrightarrow{t_\delta} \text{‘and then} \]
No Verbal Disagreement Between Two Conjunctions

If the following two conditions hold:

1. ‘∧’ is a conjunction in $L_1$ and ‘&’ is a conjunction in $L_2$, and
2. $t_1: L_1 \not\subseteq L_2$, and $t_2: L_2 \not\subseteq L_2$ are both conjunction preserving.

Then ‘∧’ and ‘&’ are equivalent in $L$. That is, in $L$, $A \land B \vdash A \& B$ and $A \& B \vdash A \land B$.
If the following two conditions hold:

1. ‘\(\land\)’ is a conjunction in \(L_1\) and ‘\(\&\)’ is a conjunction in \(L_2\), and
No Verbal Disagreement Between Two Conjunctions

If the following two conditions hold:

1. ‘\(^\land\)’ is a conjunction in \(L_1\) and ‘\(\&\)’ is a conjunction in \(L_2\), and
2. \(t_1 : L_1 \rightarrow L_\ast\), and \(t_2 : L_2 \rightarrow L_\ast\) are both conjunction preserving.

Then ‘\(^\land\)’ and ‘\(\&\)’ are equivalent in \(L_\ast\).

That is, in \(L_\ast\),

\[A \ ^\land\ B \vdash A \ & B\text{ and }A \ & B \vdash A \ ^\land\ B\]
If the following two conditions hold:

1. ‘∧’ is a conjunction in $L_1$ and ‘&’ is a conjunction in $L_2$, and
2. $t_1 : L_1 \rightarrow L_*$, and $t_2 : L_2 \rightarrow L_*$ are both conjunction preserving.

then ‘∧’ and ‘&’ are equivalent in $L_*$. 
No Verbal Disagreement Between Two Conjunctions

If the following two conditions hold:

1. ‘∧’ is a conjunction in $L_1$ and ‘&’ is a conjunction in $L_2$, and
2. $t_1 : L_1 \rightarrow L_*$, and $t_2 : L_2 \rightarrow L_*$ are both conjunction preserving.

then ‘∧’ and ‘&’ are equivalent in $L_*$. That is, in $L_*$, $A \land B \vdash A \& B$ and $A \& B \vdash A \land B$. 
No Verbal Disagreement Between Two Conjunctions

If the following two conditions hold:

1. ‘∧’ is a conjunction in $L_1$ and ‘&’ is a conjunction in $L_2$, and
2. $t_1 : L_1 \to L_*$, and $t_2 : L_2 \to L_*$ are both conjunction preserving.

then ‘∧’ and ‘&’ are equivalent in $L_*$. That is, in $L_*$, $A \land B \vdash A \land B$ and $A \land B \vdash A \land B$.

Why?
Reason as follows inside $L_*$:
Here's why

Reason as follows inside $L_*$:

\[
\begin{align*}
A \land B \vdash A \land B & \quad [\land] \\
A, B \vdash A \land B & \quad [\land] \\
A \land B \vdash A \land B & \quad [\land] \\
A \land B \vdash A \land B & \quad [\land] \\
A \land B & \vdash A \land B \\
A \land B & \vdash A \land B
\end{align*}
\]

(Since $\land$ and $\&$ are both conjunctions in $L_*$.)
If ‘∧’ and ‘&’ are equivalent, then any merely verbal disagreement between A ∧ B and A’ & B’ cannot be explained by an equivocation between ‘∧’ and ‘&’.
If ‘∧’ and ‘&’ are equivalent, then any merely verbal disagreement between $A \land B$ and $A' \land B'$ cannot be explained by an equivocation between ‘∧’ and ‘&’.

The only way to coherently assert $A \land B$ and deny $A' \land B'$ involves distinguishing $A$ and $A'$ or $B$ and $B'$.
Equivalence and Verbal Disagreements

If ‘\(^\land\)’ and ‘\(\&\)’ are equivalent, then any merely verbal disagreement between \(A \land B\) and \(A' \& B'\) cannot be explained by an equivocation between ‘\(^\land\)’ and ‘\(\&\)’.

The only way to coherently assert \(A \land B\) and deny \(A' \& B'\) involves distinguishing \(A\) and \(A'\) or \(B\) and \(B'\).

\[
\begin{align*}
A' & \land B' \vdash A' \& B' \\
B \vdash B' \quad A', B' \vdash A' \land B' & \quad [\&\uparrow] \\
A' \vdash A' \quad A', B \vdash A' \land B' \ & \quad [\text{Cut}] \\
A \vdash A' \quad A', B \vdash A' \land B' \ & \quad [\text{Cut}] \\
A, B \vdash A' \land B' \ & \quad [\land\downarrow] \\
A \land B \vdash A' \land B' 
\end{align*}
\]

If \(A/A'\) and \(B/B'\) are equivalent, so are \(A \land B\) and \(A' \& B'\).
This is not surprising… since the rules for conjunction are very strong.
This is not surprising...

... since the rules for conjunction are very strong.
Consider the debate between the intuitionist and classical logician over negation.
Consider the debate between the intuitionist and classical logician over negation.

*Dummett*: I assert $\neg\neg p$ and deny $p$: $\neg\neg p \not\vdash p$. 

Williamson: $\neg p \vdash p$. Could this be a merely verbal disagreement? Of course! There are logics in which both intuitionist and classical ‘negation’ can be distinguished. Sort of.
Consider the debate between the intuitionist and classical logician over negation.

*Dummett*: I assert $\neg \neg p$ and deny $p$: $\neg \neg p \not\vdash p$.

*Williamson*: $\neg \neg p \vdash p$. 
Consider the debate between the intuitionist and classical logician over negation.

*Dummett*: I assert \( \neg \neg p \) and deny \( p \): \( \neg \neg p \not\vdash p \).

*Williamson*: \( \neg \neg p \vdash p \).

Could *this* be a merely verbal disagreement?
Consider the debate between the intuitionist and classical logician over negation.

*Dummett*: I assert \( \neg \neg p \) and deny \( p \): \( \neg \neg p \not\vdash p \).

*Williamson*: \( \neg \neg p \vdash p \).

Could this be a merely verbal disagreement?

Of course! There are logics in which both intuitionist and classical ‘negation’ can be distinguished.
Consider the debate between the intuitionist and classical logician over negation.

*Dummett*: I assert \( \neg \neg p \) and deny \( p \): \( \neg \neg p \not\vdash p \).

*Williamson*: \( \neg \neg p \vdash p \).

Could *this* be a merely verbal disagreement?

Of course! There are logics in which both intuitionist and classical ‘negation’ can be distinguished.

*Sort of.*
Negation

When is something a *negation*?
When is something a *negation*?

**CLASSICAL LOGIC:**

\[
\frac{X \vdash A, Y}{X, \neg A \vdash Y} \quad [-\Downarrow]
\]
Negation

When is something a *negation*?

**CLASSICAL LOGIC:**

\[
\frac{X \vdash A, Y}{X, \neg A \vdash Y} \quad [\neg \downarrow]
\]

**INTUITIONIST LOGIC:**

\[
\frac{X, A \vdash}{X \vdash \neg A} \quad [\neg \downarrow]
\]

Let's call something a negation in $L$ if it satisfies at least the intuitionist negation rules. And let's say that $t : L_1 ! L_2$ preserves negation if it translates a negation in $L_1$ by a negation in $L_2$. 

Greg Restall
http://consequently.org/presentation/2015/verbal-disputes-oxford/
When is something a *negation*?

**CLASSICAL LOGIC:**

\[
\frac{X \vdash A, Y}{X, \neg A \vdash Y} \quad \text{[\(\neg \downarrow\)]} \\
\frac{X, \neg A \vdash Y}{X \vdash \neg A} \quad \text{[\(\neg \downarrow\)]}
\]

**INTUITIONIST LOGIC:**

\[
\frac{X, A \vdash}{} \quad \text{[\(\neg \downarrow\)]}
\]

Let’s call something a NEGATION in L if it satisfies *at least* the intuitionist negation rules.
Negation

When is something a *negation*?

**CLASSICAL LOGIC:**

\[
\frac{X \vdash A, Y}{X, -A \vdash Y} \quad \text{[\(-\downarrow\)]}
\]

**INTUITIONIST LOGIC:**

\[
\frac{X, A \vdash -A}{X \vdash -A} \quad \text{[\(-\downarrow\)]}
\]

Let’s call something a **NEGATION** in \(L\) if it satisfies *at least* the intuitionist negation rules.

And let’s say that \(t : L_1 \rightarrow L_2\) **PRESERVES NEGATION** if it translates a negation in \(L_1\) by a negation in \(L_2\).
No Verbal Disagreement Between Two Negations

If the following two conditions hold:

1. ‘:\' is a negation in \( L_1 \) and ‘-:\' is a negation in \( L_2 \), and
2. \( t_1 \) \( :L_1 \not= L_2 \), and \( t_2 \) \( :L_2 \not= L_2 \) are both negation preserving.

then ‘:\' and ‘-:\' are equivalent in \( L \).

That is, in \( L \), ‘:\' \( A \vdash -A \) and ‘-:\' \( -A \vdash :A \).
If the following *two* conditions hold:

1. ‘−’ is a negation in $L_1$ and ‘―’ is a negation in $L_2$, and
No Verbal Disagreement Between Two Negations

If the following *two* conditions hold:

1. ‘¬’ is a negation in $L_1$ and ‘—’ is a negation in $L_2$, and
2. $t_1 : L_1 \rightarrow L_*$, and $t_2 : L_2 \rightarrow L_*$ are both negation preserving.

That is, in $L_*$, $\vdash \neg A$ and $\neg A \vdash A$. Why?
If the following two conditions hold:

1. ‘⁻’ is a negation in $L_1$ and ‘−’ is a negation in $L_2$, and
2. $t_1 : L_1 \rightarrow L_*$, and $t_2 : L_2 \rightarrow L_*$ are both negation preserving.

then ‘⁻’ and ‘−’ are equivalent in $L_*$.
If the following two conditions hold:

1. ‘→’ is a negation in $L_1$ and ‘→’ is a negation in $L_2$, and
2. $t_1 : L_1 \rightarrow L_*$, and $t_2 : L_2 \rightarrow L_*$ are both negation preserving.

then ‘→’ and ‘→’ are equivalent in $L_*$. That is, in $L_*$, $\neg A \vdash \neg A$ and $\neg A \vdash \neg A$. 
If the following two conditions hold:

1. ‘∼’ is a negation in $L_1$ and ‘∼’ is a negation in $L_2$, and
2. $t_1 : L_1 \rightarrow L_*$, and $t_2 : L_2 \rightarrow L_*$ are both negation preserving.

then ‘∼’ and ‘∼’ are equivalent in $L_*$. That is, in $L_*$, $\neg A \vdash \neg \neg A$ and $\neg \neg A \vdash \neg A$.

Why?
Reason as follows inside $L_*$:

\[
\begin{align*}

\neg A &\vdash \neg A & &\neg A &\vdash \neg A & &\neg A &\vdash \neg A \\

\neg A, A &\vdash & &\neg A, A &\vdash & &\neg A &\vdash \neg A \\

\neg A &\vdash \neg A & &\neg A &\vdash \neg A \\

\end{align*}
\]

It follows that any disagreement, where one asserts $\neg A$ and the other denies $\neg A$ (or *vice versa*), must resolve into a disagreement over $A$. 
If ‘∼’ and ‘¬’ are equivalent, then any merely verbal disagreement between ∼A and ¬A’ cannot be explained by an equivocation between the two negations.

The only way to coherently assert ∼A and deny ¬A’ involves distinguishing A and A’. 

\[
\begin{align*}
\neg A & \vdash \neg A \\
\neg A, A & \vdash [\neg \uparrow] A \vdash A' \\
\neg A, A' & \vdash [\text{Cut}] \\
& \vdash \neg A \vdash \neg A'
\end{align*}
\]
What options are there for disagreement?

- Disagreement over the consequence relation ‘⊢’ (*pluralism*).

- The classical logician thinks the intuitionist is mistaken to take ‘⊢’ to be so weak, or the intuitionist thinks that the classical logician is mistaken to take ‘¬’ to be so strong.
Can we have merely verbal disagreement about ‘exists’?
Can we have merely verbal disagreement about ‘exists’?

Can we have merely verbal disagreement about ‘(∃x)’?
Ontological Relativity

Can we have merely verbal disagreement about ‘exists’?

Can we have merely verbal disagreement about ‘(∃x)’?

Surely!
Can we have merely verbal disagreement about ‘exists’?

Can we have merely verbal disagreement about ‘(∃x)’?

Surely! Take *multi-sorted* first order logic. α says that there are numbers ((∃x)Nx). δ denies it (¬(∃x)Nx). Can we make this difference *merely verbal*? While respecting some of the semantics of (∃x)?
Can we have merely verbal disagreement about ‘exists’?

Can we have merely verbal disagreement about ‘(∃x)’?

Surely! Take multi-sorted first order logic. α says that there are numbers ((∃x)Nx). δ denies it (¬(∃x)Nx). Can we make this difference merely verbal? While respecting some of the semantics of (∃x)?

Translate into a vocabulary with two quantifiers and two two domains: D₁ and D₂ with two quantifiers (∃₁x) and (∃₂x) ranging over each. Let N have a non-empty extension on D₁ but an empty one on D₂. Both α and δ can happily endorse (∃₁x)Nx and deny (∃₂x)Nx and be done with it.
Can we have merely verbal disagreement about ‘exists’?

Surely! Take multi-sorted first order logic. $\alpha$ says that there are numbers $((\exists x)\, N x)$. $\delta$ denies it $(\neg(\exists x)\, N x)$. Can we make this difference merely verbal? While respecting some of the semantics of $(\exists x)$?

Translate into a vocabulary with two quantifiers and two two domains: $D_1$ and $D_2$ with two quantifiers $(\exists_1 x)$ and $(\exists_2 x)$ ranging over each. Let $N$ have a non-empty extension on $D_1$ but an empty one on $D_2$. Both $\alpha$ and $\delta$ can happily endorse $(\exists_1 x)\, N x$ and deny $(\exists_2 x)\, N x$ and be done with it.

Isn’t this a merely verbal disagreement over what exists?
Not so fast...

Perhaps there is scope for the same behaviour as with conjunction and negation. Consider more closely what might be involved in being an existential quantifier, and a translation preserving it.

\[ X; A(v) \vdash Y = X; (\exists x) A(x) \vdash Y \] (Where \( v \) is not free in \( X \) and \( Y \).)

This is what it takes to be an existential quantifier in \( L \).

Greg Restall

http://consequently.org/presentation/2015/verbal-disputes-oxford/
Not so fast...

Perhaps there is scope for the same behaviour as with conjunction and negation.
Perhaps there is scope for the same behaviour as with conjunction and negation. Consider more closely what might be involved in being an existential quantifier, and a translation preserving it.
Perhaps there is scope for the same behaviour as with conjunction and negation. Consider more closely what might be involved in being an existential quantifier, and a translation preserving it.

\[
\frac{X, A(\nu) \vdash Y}{X, (\exists x)A(x) \vdash Y} \text{[}\exists\top\text{]} \\
\text{(Where } \nu \text{ is not free in } X \text{ and } Y.\text{)}
\]

This is what it takes to be an existential quantifier in L.
Existential Quantifier Collapse

\[
\begin{align*}
(\exists_2 x)A(x) & \vdash (\exists_2 x)A(x) & [\exists_2 \uparrow] \\
A(v) & \vdash (\exists_2 x)A(x) & [\exists_1 \downarrow] \\
(\exists_1 x)A(x) & \vdash (\exists_2 x)A(x) & \text{[3.1]} \\
(\exists_2 x)A(x) & \vdash (\exists_1 x)A(x) & \text{[3.2]} \\
A(v) & \vdash (\exists_1 x)A(x) & \text{[3.3]} \\
(\exists_1 x)A(x) & \vdash (\exists_2 x)A(x) & \text{[3.4]}
\end{align*}
\]
Existential Quantifier Collapse

\[
\frac{(\exists_2 x) A(x) \vdash (\exists_2 x) A(x)}{A(v) \vdash (\exists_2 x) A(x)} \quad \text{[}\exists_2 \uparrow\text{]}
\]

\[
\frac{(\exists_1 x) A(x) \vdash (\exists_2 x) A(x)}{(\exists_1 x) A(x) \vdash (\exists_2 x) A(x)} \quad \text{[}\exists_1 \downarrow\text{]}
\]

\[
\frac{(\exists_1 x) A(x) \vdash (\exists_1 x) A(x)}{A(v) \vdash (\exists_1 x) A(x)} \quad \text{[}\exists_2 \downarrow\text{]}
\]

\[
\frac{(\exists_2 x) A(x) \vdash (\exists_1 x) A(x)}{(\exists_2 x) A(x) \vdash (\exists_1 x) A(x)} \quad \text{[}\exists_2 \uparrow\text{]}
\]

If the term \( v \) appropriate to \( [\exists_1 \uparrow] \) also applies in \( [\exists_2 \downarrow] \), and vice versa, then indeed, the two quantifiers collapse.
Coordination on *terms* brings coordination on \((\exists x)\)

If the following *three* conditions hold:

1. ‘\((\exists_1 x)\)’ is an existential quantifier in \(L_1\) and ‘\((\exists_2 x)\)’ is an existential quantifier in \(L_2\), and

2. \(t_1 : L_1 \rightarrow L_*\), and \(t_2 : L_2 \rightarrow L_*\), are both *existential quantifier preserving*, and

3. In \(L_*\), some fresh term \(v\) is *appropriate for both* \((\exists_1 x)\) and \((\exists_2 x)\)

then \((\exists_1 x)\) and \((\exists_2 x)\) are *equivalent* in \(L_*\), in that in \(L_*\) we have

\((\exists_1 x)A \vdash (\exists_2 x)A\) and \((\exists_2 x)A \vdash (\exists_1 x)A\).
Coordination on terms brings coordination on \((\exists x)\)

If the following three conditions hold:

1. ‘\((\exists_1 x)\)’ is an existential quantifier in \(L_1\) and ‘\((\exists_2 x)\)’ is an existential quantifier in \(L_2\), and

2. \(t_1 : L_1 \rightarrow L_*\), and \(t_2 : L_2 \rightarrow L_*\), are both existential quantifier preserving, and

3. In \(L_*\), some fresh term \(v\) is appropriate for both \((\exists_1 x)\) and \((\exists_2 x)\)

then \((\exists_1 x)\) and \((\exists_2 x)\) are equivalent in \(L_*\), in that in \(L_*\) we have

\[(\exists_1 x)A \vdash (\exists_2 x)A\] and \[(\exists_2 x)A \vdash (\exists_1 x)A.\]
The appropriateness condition for eigenvariables (demonstratives, terms) is *grammatical*. It doesn’t force agreement on *what exists*.
It's important to recognise what this is not.

The appropriateness condition for eigenvariables (demonstratives, terms) is *grammatical*. It doesn’t force agreement on *what exists*.

You could coherently be a *monist* and argue with someone with a more conventional ontology—with the *same* quantifiers—provided that you both took the same terms (demonstratives, eigenvariables, whatever) to be in order for that quantifier.
The appropriateness condition for eigenvariables (demonstratives, terms) is \textit{grammatical}. It doesn’t force agreement on \textit{what exists}.

You could coherently be a \textit{monist} and argue with someone with a more conventional ontology—with the \textit{same} quantifiers—provided that you both took the same terms (demonstratives, eigenvariables, whatever) to be in order for that quantifier.

You \textit{don’t} need to take these terms to \textit{refer} to (or range over) the same things in any substantial sense.
A Monist arguing with a Pluralist (agreeing on terms)

MONIST:  
\[ (\forall x)(\forall y)x = y \]

PLURALIST:  
\[ (\forall x)(\forall y)x \neq y \]
A Monist arguing with a Pluralist (agreeing on terms)

**MONIST:**
- $(\forall x)(\forall y)x = y$

**PLURALIST:**
- $(\exists x)(\exists y)x \neq y$
A *Monist* arguing with a *Pluralist* (agreeing on terms)

**MONIST:**
- $(\forall x)(\forall y) x = y$

**PLURALIST:**
- $(\exists x)(\exists y) x \neq y$
- $(\exists y) a \neq y$
A Monist arguing with a Pluralist (agreeing on terms)

**MONIST:**
- $(\forall x)(\forall y) x = y$
- $(\forall y) a = y$

**PLURALIST:**
- $(\exists x)(\exists y) x \neq y$
- $(\exists y) a \neq y$
A Monist arguing with a Pluralist (agreeing on terms)

**MONIST:**
- $$(\forall x)(\forall y)x = y$$
- $$(\forall y)a = y$$

**PLURALIST:**
- $$(\exists x)(\exists y)x \neq y$$
- $$(\exists y)a \neq y$$
- $$a \neq b$$
A Monist arguing with a Pluralist (agreeing on terms)

MONIST:
- \((\forall x)(\forall y)x = y\)
- \((\forall y)a = y\)
- \(a = b\)

PLURALIST:
- \((\exists x)(\exists y)x \neq y\)
- \((\exists y)a \neq y\)
- \(a \neq b\)

http://consequently.org/presentation/2015/verbal-disputes-oxford/
A Monist arguing with a Pluralist (agreeing on terms)

**MONIST:**
- $(\forall x)(\forall y)x = y$
- $(\forall y)a = y$
- $a = b$

**PLURALIST:**
- $(\exists x)(\exists y)x \neq y$
- $(\exists y)a \neq y$
- $a \neq b$
- $Fa, \neg Fb$
A Monist arguing with a Pluralist (agreeing on terms)

**MONIST:**
- $(\forall x)(\forall y)x = y$
- $(\forall y)a = y$
- $a = b$
- $Fa, Fb$

**PLURALIST:**
- $(\exists x)(\exists y)x \neq y$
- $(\exists y)a \neq y$
- $a \neq b$
- $Fa, \neg Fb$
A Monist arguing with a Pluralist (agreeing on terms)

**MONIST:**
- $(\forall x)(\forall y)x = y$
- $(\forall y)a = y$
- $a = b$
- $Fa, Fb$

**PLURALIST:**
- $(\exists x)(\exists y)x \neq y$
- $(\exists y)a \neq y$
- $a \neq b$
- $Fa, \neg Fb$
A Monist arguing with a Pluralist (disagreeing on terms)

If the pluralist had argued instead:

- $(\exists x)(\exists y)x \neq y$, because
If the pluralist had argued instead:

- $(\exists x)(\exists y) x \neq y$, because
- $\wedge \neq \supset$, since
If the pluralist had argued instead:

- $(\exists x)(\exists y)x \neq y$, because
- $\wedge \neq \supset$, since
- $\wedge$ is commutative and $\supset$ is not,
If the pluralist had argued instead:

- $(\exists x)(\exists y)x \neq y$, because
- $\land \neq \supset$, since
- $\land$ is commutative and $\supset$ is not,

It’s fair for the monist (or anyone else) to agree
A Monist arguing with a Pluralist (disagreeing on terms)

If the pluralist had argued instead:
- $(\exists x)(\exists y)x \neq y$, because
- $\land \neq \supset$, since
- $\land$ is commutative and $\supset$ is not,

It’s fair for the monist (or anyone else) to agree
- $\land$ is commutative, and $\supset$ is not.
A Monist arguing with a Pluralist (disagreeing on terms)

If the pluralist had argued instead:

- $(\exists x)(\exists y)x \neq y$, because
- $\wedge \neq \Pp$, since
- $\wedge$ is commutative and $\Pp$ is not,

It’s fair for the monist (or anyone else) to agree

- $\wedge$ is commutative, and $\Pp$ is not

But to not take these to be predications of the form $Fa$ and $\neg Fb$, and so, to not be appropriate to substitute into the quantifier.
Can we have merely verbal disagreement about ‘possibility’?
Can we have merely verbal disagreement about ‘possibility’?

Can we have merely verbal disagreement about ‘◊’?
Can we have merely verbal disagreement about ‘possibility’?

Can we have merely verbal disagreement about ‘◊’?

Surely!
Can we have merely verbal disagreement about ‘possibility’?

Can we have merely verbal disagreement about ‘◊’?

Surely! Take multi-modal logic. ◊₁ ranges over possible worlds; ◊₂ ranges over times.
Can we have merely verbal disagreement about ‘possibility’?

Can we have merely verbal disagreement about ‘◊’?

Surely! Take multi-modal logic. ◊₁ ranges over possible worlds; ◊₂ ranges over times.

Isn’t this a merely verbal disagreement over what possible?
Let’s consider more closely what might be involved in *possibility preservation*.

\[
A \vdash | X \vdash Y | \Delta \\
\frac{}{X, \Diamond A \vdash Y | \Delta} \quad [\Diamond \bot]
\]

The separated sequents indicate positions in which assertions and denials are made in different *zones* of a discourse.
Not so fast...

Let’s consider more closely what might be involved in \textit{possibility preservation}.

\[
\frac{A \vdash X \vdash Y | \Delta}{\vdash X, \Diamond A \vdash Y | \Delta} \text{ [◊□]}
\]

The separated sequents indicate positions in which assertions and denials are made in different \textit{zones} of a discourse.

For details, see


If the zone appropriate to $[\diamond_1 \uparrow]$ also applies in $[\diamond_2 \downarrow]$, and vice versa then indeed, the two operators collapse.
If the following *three* conditions hold:

1. ‘◊₁’ is an possibility in L₁ and ‘◊₂’ is an possibility in L₂, and
2. t₁ : L₁ → L*, and t₂ : L₂ → L*, are both *possibility preserving*, and
3. In L*, a zone is *appropriate* for ◊₁ iff it is appropriate for ◊₂

then ◊₁ and ◊₂ are *equivalent* in L*, in that in L* we have ◊₁A ⊢ ◊₂A and ◊₂A ⊢ ◊₁A.
Coordination on zones brings coordination on ◊

If the following three conditions hold:

1. ‘◊₁’ is an possibility in L₁ and ‘◊₂’ is an possibility in L₂, and
2. t₁ : L₁ → L*, and t₂ : L₂ → L*, are both possibility preserving, and
3. In L*, a zone is appropriate for ◊₁ iff it is appropriate for ◊₂

then ◊₁ and ◊₂ are equivalent in L*, in that in L* we have ◊₁A ⊨ ◊₂A and ◊₂A ⊨ ◊₁A.
It's important to recognise what this is not

The appropriateness condition for zones is *dialogical*. It doesn’t force agreement on what is possible.
The appropriateness condition for zones is *dialogical*. It doesn't force agreement on *what is possible*.

You could coherently be a *modal fatalist* and argue with someone with a more conventional modal views—with the *same* modal operators, provided that you both took the same zones to be in order.
It's important to recognise what this is not

The appropriateness condition for zones is dialogical. It doesn’t force agreement on what is possible.

You could coherently be a modal fatalist and argue with someone with a more conventional modal views—with the same modal operators, provided that you both took the same zones to be in order.

(You don’t need to take the same things to hold in each zone.)
THE UPSHOT
The more you want from a translation, the fewer translations you have, and the fewer ways there are to settle disputes as merely verbal.
Upshot #1: The Power of Keeping Some Things Fixed

The more you want from a translation, the fewer translations you have, and the fewer ways there are to settle disputes as merely verbal.

And the more chance you have to *locate* that dispute in some particular part of your vocabulary.
It’s one thing to think of a logical concept as something satisfying a set of *axioms*.
It’s one thing to think of a logical concept as something satisfying a set of axioms.

But that is *cheap*. Defining rules are *more powerful*.

And defining rules are natural, given the conception of logical constants as topic neutral, and definable in general terms.
Upshot #3: Generality Comes in Degrees

1. Propositional connectives: sequents alone.
Upshot #3: Generality Comes in Degrees

1. Propositional connectives: sequents alone.

Using this structure to define the behaviour of a logical concepts allows for them to be preserved in translation and used as a fixed point in the midst of disagreement.
THANK YOU!

http://consequently.org/presentation/2015/verbal-disputes-oxford/

@consequently on Twitter