## New Work for a (Formal) Theory of Grounds

#### Greg Restall



#### Melbourne logic seminar $\cdot$ 14 december 2018

#### My Aim

## Grounds

#### Epistemic

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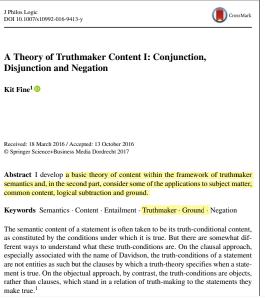
#### The epistemic significance of valid inference

**Dag Prawitz** 

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Abstract The traditional picture of logic takes it for granted that "valid arguments have a fundamental epistemic significance,", but neither model theory nor traditional proof theory dealing with formal system has been able to give an account of this significance. Since valid arguments as usually understood do not in general have any epistemic significance, the problem is to explain how and why we can neverthless use them sometimes to acquire knowledge. It is suggested that we should distinguish between arguments and acts of inferences and that we have to reconsider the latter notion to arrive at the desired explanation. More precisely, the notions should be developed so that the following relationship holds: one gets in possession of a ground for a conclusion by inferring it from premises for which one already has grounds, provided that the inference in question is valid. The paper proposes explications of the concepts of ground and deductively valid inference is seen as a special case of deductive validity, but does not add anything as far as epistemic significance is concerned—it resides already in the deductively valid inferences.

#### Metaphysical



My Plan

### Desiderata

### Why These Desiderata?

Two Models

# DESIDERATA

#### 1. Grammar

#### g is a ground for A.

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#### g is a ground for A.

#### g is a ground *against* A.

#### 2. Deduction

A derivation of X > A, Ygives us a systematic way to construct a ground for A from grounds for each member of X and grounds *against* each member of Y.

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A derivation of  $X \succ A, Y$ gives us a systematic way to construct a ground for A from grounds for each member of X and grounds against each member of Y.

A derivation of X,  $B \succ Y$ gives us a systematic way to construct a ground *against* B from grounds *for* each member of X and grounds *against* each member of Y.

#### 3. Interpretation

#### Epistemic

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#### Epistemic

#### Metaphysical

#### 4. Grasp

#### Grounds are the kinds of things we can possess.

#### 5. Hyperintensionality

#### Not every ground is a ground for every tautology.

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#### Not every ground is a ground for every tautology.

#### A ground for A need not also be a ground for each logical consequence of A.

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# WHY THESE DESIDERATA?

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# We not only learn *that* there are grounds for A, we *obtain* grounds for A.

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An account in terms of grounds does.

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#### In the BHK interpretation of intuitionist logic, the constructivist *has* a theory of grounds of this general form.

I'd like to know if this is possible for the classical logician.

In the rest of this talk I'll present two *models*, which both show how the desiderata can be jointly satisfied. In the rest of this talk I'll present two *models*, which both show how the desiderata can be jointly satisfied.

These models are merely *models*. I don't offer them as accounts of what grounds *are*.

# TWO MODELS

#### **Classical Sequents**

# $\boldsymbol{X}\succ\boldsymbol{Y}$

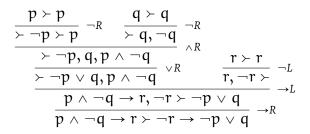
#### where X and Y are finite multisets of formulas

#### **Classical Sequent Calculus**

$$\begin{array}{ccc} X,p\succ p,Y\left[Id\right] & \displaystyle \frac{X\succ A,Y & X,A\succ Y}{X\succ Y} \ Cut \\ & \displaystyle \frac{X,A,A\succ Y}{X,A\succ Y} \ WL & \displaystyle \frac{X\succ A,A,Y}{X\succ A,Y} \ WR \\ & \displaystyle \frac{X\succ A,Y}{X,A\succ Y} \ WL & \displaystyle \frac{X\succ A,A,Y}{X\succ A,Y} \ WR \\ & \displaystyle \frac{X\succ A,Y}{X,\neg A\succ A,Y} \ \neg L & \displaystyle \frac{X,A\succ Y}{X\succ \neg A,Y} \ \neg R \\ & \displaystyle \frac{X,A,B\succ Y}{X,A\land B\succ Y} \ \land L & \displaystyle \frac{X\succ A,Y & X'\succ B,Y'}{X,X'\succ A\land B,Y,Y'} \ \land R \\ & \displaystyle \frac{X,A\succ Y & X',B\succ Y'}{X,X',A\lor B\succ Y,Y'} \ \lor L & \displaystyle \frac{X\succ A,B,Y}{X\succ A\lor B,Y} \ \lor R \\ & \displaystyle \frac{X\succ A,Y & X',B\succ Y'}{X,X',A\to B,Y,Y'} \rightarrow L & \displaystyle \frac{X,A\succ B,Y}{X\succ A\to B,Y} \rightarrow R \end{array}$$

#### Derivations





#### Positions

#### A *position* is a pair [L : R] of *sets* of formulas.

(L and R need not be finite.)

#### Derivations for

Given a position [L : R]a derivation  $\delta$  of  $X \succ Y$ is a derivation for  $L \rhd R$ iff  $X \subseteq L$  and  $Y \subseteq R$ .

Note: This extends the subset relation to relate finite *multisets* to sets.  $X \subseteq L$  iff each member of X (of whatever multiplicity) is also member of L.

#### Derivations for

# $\frac{L \rhd A, R \quad L, A \rhd R}{L \rhd R} \ {\it Cut}$

#### **Available Positions**

# A position [L : R] is *available* iff there is no derivation for L > R.

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  - Example: a partition [T : F] on the atoms, i.e. a boolean evaluation.
  - *Example*: a set [L : R] of formulas L immediately given as true in experience, and R immediately given as false in experience.
- Given a basis [L : R], a ground for A is a derivation for L ▷ A, R; and a ground against A is a derivation for L, A ▷ R.

#### The Motivating Idea

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A ground *against* A shows how A is ruled out by the basis.

$$\label{eq:constraint} \begin{split} \text{Take a boolean evaluation basis} \ [\mathsf{T}:\mathsf{F}] \ \text{where} \ p \in \mathsf{T} \ \text{and} \ q \in \mathsf{F}. \\ \\ \text{Here are three different grounds for} \ (p \lor \neg p) \lor \neg q. \end{split}$$

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$$\frac{p \succ p}{p \succ p \lor \neg p} \lor R_1 \\ p \succ (p \lor \neg p) \lor \neg q} \lor R_2$$

Take a boolean evaluation basis [T : F] where  $p \in T$  and  $q \in F$ . Here are three different grounds for  $(p \lor \neg p) \lor \neg q$ .

$$\frac{p \succ p}{p \succ p \lor \neg p} \lor_{R_1} \qquad \frac{q \succ q}{\succ \neg q, q} \lnot_{R_2} \qquad \frac{q \succ q}{\succ \neg q, q} \lor_{R_1} \lor_{R_2}$$

Take a boolean evaluation basis [T : F] where  $p \in T$  and  $q \in F$ . Here are three different grounds for  $(p \lor \neg p) \lor \neg q$ .  $p \succ p$   $q \succ q$ 

$$\frac{\frac{p \succ p}{p \succ p \lor \neg p} \lor R_1}{p \succ (p \lor \neg p) \lor \neg q} \lor R_2 \qquad \frac{\frac{q \succ q}{\succ \neg q, q} \neg R}{\succ (p \lor \neg p) \lor \neg q, q} \lor R_1$$

$$\frac{\frac{p \succ p}{\succ p \lor \neg p} \neg R}{\succ (p \lor \neg p) \lor q} \lor R_1$$

#### $\delta$ is a ground for A.

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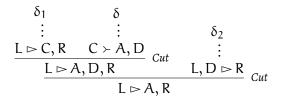
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## $\delta$ is a ground against A. $\delta$ is a derivation for [L, A : R].

If  $\delta$  is a derivation of  $C \succ A$ , D and  $\delta_1$  is a ground for C, and  $\delta_2$  is a ground against D we can construct a ground for A like this:

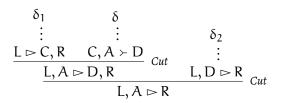
If  $\delta$  is a derivation of  $C \succ A$ , D and  $\delta_1$  is a ground for C, and  $\delta_2$  is a ground against D we can construct a ground for A like this:



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we can construct a ground against A like this:

$$\frac{\begin{array}{cccc}
\delta_{1} & \delta \\
\vdots & \vdots & \delta_{2} \\
\underline{L \rhd C, R & C, A \succ D} \\
\underline{L, A \rhd D, R} & Cut & \vdots \\
\underline{L, A \rhd R} & L, D \rhd R \\
\end{array}}$$
Cut

(*This generalises*: a derivation of  $X \succ A$ , Y [of  $X, A \succ Y$ ] can be used to convert grounds for each member of X and against each member of Y into grounds for A [against A].)

#### 3. Interpretation

#### We can think of a boolean valuation basis [L : R] as a description of a *world*, for a model of metaphysical grounding.

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> We can think of a limited basis [L : R] as modelling an *evidence base*, for a model of epistemic grounding.

#### 4. Grasp

A ground is a derivation. It is finite, and so it can be grasped (at least, in principle).

#### 5. Hyperintensionality

#### A ground for A is a derivation for $L \triangleright A$ , R.

It is not a ground for any other formula outside the basis [L : R].

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It is not a ground for any other formula outside the basis [L : R].

Each valid formula has a ground (its derivation).

If  $\delta$  is a ground for A, and A entails B, then we can ground B using  $\delta$ *cut with* the derivation of B.

• A ground for  $A \rightarrow B$  can be seen as a function from grounds for A to grounds for B.

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$$\frac{\begin{array}{ccc} \delta \\ \vdots \\ \underline{L \rhd A \rightarrow B, R} \\ \underline{A, A \rightarrow B \succ B} \\ \underline{L, A \rhd B, R} \\ \underline{L \rhd B, R} \\ \hline \begin{array}{c} L \\ \underline{L \rhd B, R} \end{array} \xrightarrow{b \leftarrow B} Cut \\ \underline{L \rhd A, R} \\ Cut \\ \hline \end{array} Cut$$

- A ground for A  $\wedge$  B can be seen consisting of a ground for A and a ground for B.

• A ground for  $A \land B$  can be seen consisting of a ground for A and a ground for B.

$$\frac{\substack{\delta \\ \vdots \\ L \rhd A, R \\ L \rhd A \land B, R} \land R}{L \rhd B, R}$$

• A ground for  $A \land B$  can be seen consisting of a ground for A and a ground for B.

$$\frac{\delta}{\vdots} \qquad \frac{\delta'}{\vdots} \qquad \frac{\lambda}{\Box \rhd A, R} \qquad \frac{\lambda}{\Box \rhd A, B, R} \land R}{\Box \rhd A, B, R} \land R$$

$$\frac{\delta}{\vdots} \qquad \frac{A, B \succ A}{\Box \rhd A, R} \land R \qquad \frac{\delta}{\Box t} \qquad \frac{\delta}{\Box \rhd A, B, R} \qquad \frac{A, B \succ B}{A \land B \succ B} \land R}{\Box t} \qquad \frac{\Delta, B \succ B}{\Box t} \qquad \frac{\Delta, R}{\Box t} \qquad \frac{\Delta, B \succ B}{\Box t} \qquad \frac{\Delta, R}{\Box t} \qquad \frac{\Delta, B \succ B}{\Box t} \qquad \frac{\Delta, R}{\Box t} \qquad \frac{\Delta, B \succ B}{\Box t} \qquad \frac{\Delta, R}{\Box t} \qquad \frac{\Delta, B \succ B}{\Box t} \qquad \frac{\Delta, B \succ B}{\Box t} \qquad \frac{\Delta, R}{\Box t} \qquad \frac{\Delta, B \succ B}{\Box t} \qquad \frac{\Delta, R}{\Box t} \qquad \frac{\Delta, B \succ R}{\Box t} \qquad \frac{\Delta, B \leftarrow R}{\Box t} \qquad$$

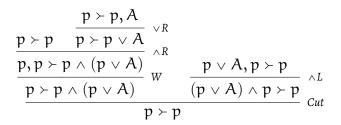
• A ground for  $A \land B$  can be seen consisting of a ground for A and a ground for B.

$$\frac{\delta}{1:} \qquad \frac{\delta'}{1:} \qquad \frac{\lambda}{L \rhd A, R} \qquad \frac{\lambda}{L \rhd A, B, R} \land R \qquad \frac{\delta}{1:} \qquad \frac{\lambda}{L \rhd A, B, R} \land R \qquad \frac{\delta}{1:} \qquad \frac{\lambda}{L \rhd A, B, R} \qquad \frac{\delta}{L \rhd A, B, R} \qquad \frac{\delta}{L \rhd A, B, R} \qquad \frac{\lambda}{A \land B \succ B} \land R \qquad \frac{\lambda}{L \rhd B, R} \qquad \frac{\lambda}{L \varsigma B, R}$$

Note: pairing grounds for A and grounds for B into a ground for  $A \land B$ , and then extracting a ground for A doesn't return the original ground for A, but a different ground.

Model 1 satisfies our desiderata, but there are many *many* different grounds which do no epistemic or metaphysical *work*.

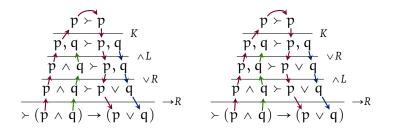
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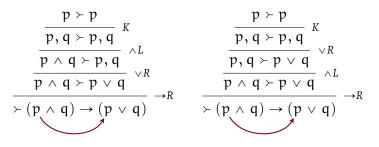
(There's a different one of these for each formula A.)

$$\frac{\frac{p \succ p}{p, q \succ p, q} K}{\frac{p \land q \succ p, q}{p \land q \succ p, q} \land L} \qquad \qquad \frac{\frac{p \succ p}{p, q \succ p, q} K}{\frac{p, q \succ p, q}{p \land q \succ p \lor q} \land L} \\ \xrightarrow{\frac{p, q \succ p \lor q}{p \land q \succ p \lor q} \land R} \qquad \qquad \frac{\frac{p, q \succ p \lor q}{p \land q \succ p \lor q} \land L}{\frac{p, q \succ p \lor q}{p \land q \succ p \lor q} \land L} \rightarrow R$$

(These derivations differ in ways that don't matter for how  $(p \land q) \rightarrow (p \lor q)$  is grounded.)

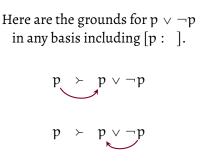


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## Given a basis [L : R], a ground for A is a proof term of a derivation for L ▷ A, R. and a ground against A is a proof term of a derivation for L, A ▷ R.



 $p \succ p \lor \neg p$ 

## There are many fewer *proof term* grounds. Each different ground represents a different way for formula to be grounded in the basis.

- Proof term grounds allow for only one ground for (or against) each atomic formula. Is this appropriate? How can this restriction best be lifted?
- Are there other models of this general shape?
- What is an appropriate basis for a metaphysical (total) interpretation for grounds for sentences in predicate logic?

# THANK YOU!

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