New Work for a (Formal) Theory of Grounds

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Grounds

Epistemic

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The epistemic significance of valid inference

Dag Prawitz

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Abstract The traditional picture of logic takes it for granted that "valid arguments have a fundamental epistemic significance", but neither model theory nor traditional proof theory dealing with formal system has been able to give an account of this significance. Since valid arguments as usually understood do not in general have any epistemic significance, the problem is to explain how and why we can nevertheless use them sometimes to acquire knowledge. It is suggested that we should distinguish between arguments and acts of inferences and that we have to reconsider the latter notion to arrive at the desired explanation. More precisely, the notions should be developed so that the following relationship holds: one gets in possession of a ground for a conclusion by inferring it from premisses for which one already has grounds, provided that the inference in question is valid. The paper proposes explications of the concepts of ground and deductively valid inference so that this relationship holds as a conceptual truth. Logical validity of inference is seen as a special case of deductive validity, but does not add anything as far as epistemic significance is concerned—it resides already in the deductively valid inferences.

Metaphysical

J Philos Logic
DOI 10.1007/s10992-016-9413-y

CrossMark

A Theory of Truthmaker Content I: Conjunction, Disjunction and Negation

Kit Fine[1]

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Abstract I develop a basic theory of content within the framework of truthmaker semantics and, in the second part, consider some of the applications to subject matter, common content, logical subtraction and ground.

Keywords Semantics · Content · Entailment · Truthmaker · Ground · Negation

The semantic content of a statement is often taken to be its truth-conditional content, as constituted by the conditions under which it is true. But there are somewhat different ways to understand what these truth-conditions are. On the clausal approach, especially associated with the name of Davidson, the truth-conditions of a statement are not entities as such but the clauses by which a truth-theory specifies when a statement is true. On the objectual approach, by contrast, the truth-conditions are objects, rather than clauses, which stand in a relation of truth-making to the statements they make true.[1]

Desiderata

Why *These* Desiderata?

Two Models

DESIDERATA

g is a ground *for* A.

g is a ground *for* A.

g is a ground *against* A.

A derivation of $X \succ A, Y$
gives us a systematic way
to construct a ground *for* A
from grounds *for* each member of X
and grounds *against* each member of Y.

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A derivation of $X, B \succ Y$
gives us a systematic way
to construct a ground *against* B
from grounds *for* each member of X
and grounds *against* each member of Y.

Epistemic

3. Interpretation

Epistemic | Metaphysical

Grounds are the kinds of things we can *possess*.

Not every ground is a ground for every tautology.

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A ground for A need not also be
a ground for each logical consequence of A.

6. Structure

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- A ground for $A \wedge B$ can be seen as consisting of a ground for A and ground for B.

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- A ground for $\neg A$ can be obtained from a ground against A.
- A ground against $\neg A$ can be obtained from a ground for A.

WHY *THESE* DESIDERATA?

We'd have an account of what we gain in obtaining a proof of A.

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in obtaining a proof of A.

We not only learn *that* there are grounds for A,
we *obtain* grounds for A.

To Account for the **Direction** of Deduction

I see a derivation of $X \succ Y$ as showing that *asserting* each member of X and *denying* each member of Y involves a clash.

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An account in terms of grounds *does*.

In the BHK interpretation of intuitionist logic,
the constructivist *has* a theory of grounds
of this general form.

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the constructivist *has* a theory of grounds
of this general form.

I'd like to know if this is possible for the classical logician.

In the rest of this talk I'll present two *models*, which both show how the desiderata can be jointly satisfied.

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These models are merely *models*. I don't offer them as accounts of what grounds *are*.

TWO MODELS

$$X \succ Y$$

where X and Y are finite multisets of formulas

Classical Sequent Calculus

$$X, p \succ p, Y \; [Id] \qquad \frac{X \succ A, Y \quad X, A \succ Y}{X \succ Y}\, Cut$$

$$\frac{X, A, A \succ Y}{X, A \succ Y}\, WL \qquad \frac{X \succ A, A, Y}{X \succ A, Y}\, WR$$

$$\frac{X \succ A, Y}{X, \neg A \succ A, Y}\, \neg L \qquad \frac{X, A \succ Y}{X \succ \neg A, Y}\, \neg R$$

$$\frac{X, A, B \succ Y}{X, A \wedge B \succ Y}\, \wedge L \qquad \frac{X \succ A, Y \quad X' \succ B, Y'}{X, X' \succ A \wedge B, Y, Y'}\, \wedge R$$

$$\frac{X, A \succ Y \quad X', B \succ Y'}{X, X', A \vee B \succ Y, Y'}\, \vee L \qquad \frac{X \succ A, B, Y}{X \succ A \vee B, Y}\, \vee R$$

$$\frac{X \succ A, Y \quad X', B \succ Y'}{X, X', A \to B \succ Y, Y'}\, {\to} L \qquad \frac{X, A \succ B, Y}{X \succ A \to B, Y}\, {\to} R$$

$$\dfrac{\dfrac{p \succ p}{\succ p, \neg p}\;\neg R}{\succ p \vee \neg p}\;\vee R \qquad\qquad \dfrac{\dfrac{p \succ p}{p, \neg p \succ}\;\neg L}{p \wedge \neg p \succ}\;\wedge L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{p \succ p}{\succ \neg p \succ p}\;\neg R \qquad \dfrac{q \succ q}{\succ q, \neg q}\;\neg R}{\succ \neg p, q, p \wedge \neg q}\;\wedge R}{\succ \neg p \vee q, p \wedge \neg q}\;\vee R \qquad \dfrac{r \succ r}{r, \neg r \succ}\;\neg L}{p \wedge \neg q \to r, \neg r \succ \neg p \vee q}\;\to L}{p \wedge \neg q \to r \succ \neg r \to \neg p \vee q}\;\to R$$

A *position* is a pair $[L : R]$ of *sets* of formulas.

(L and R need not be finite.)

Given a position $[L : R]$

a derivation δ *of* $X \succ Y$

is a derivation *for* $L \rhd R$

iff $X \subseteq L$ and $Y \subseteq R$.

Note: This extends the subset relation
to relate finite *multisets* to sets.
$X \subseteq L$ iff each member of X
(of whatever multiplicity)
is also member of L.

Derivations *for*

$$\frac{L \rhd A, R \qquad L, A \rhd R}{L \rhd R}\ \mathit{Cut}$$

Available Positions

A position $[L : R]$ is *available* iff there is no derivation for $L \rhd R$.

Model 1

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Model 1

- A *basis* $[L : R]$ is an available position.
  - *Example*: a partition $[T : F]$ on the atoms, i.e. a boolean evaluation.
  - *Example*: a set $[L : R]$ of formulas L immediately given as true in experience, and R immediately given as false in experience.
- Given a basis $[L : R]$, a ground for A is a derivation for $L \rhd A, R$; and a ground against A is a derivation for $L, A \rhd R$.

A ground *for* A shows how A is ruled in by the basis.

A ground *for* A shows how A is ruled in by the basis.

A ground *against* A shows how A is ruled out by the basis.

Example

Take a boolean evaluation basis $[T : F]$ where $p \in T$ and $q \in F$.

Here are three different grounds for $(p \vee \neg p) \vee \neg q$.

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Take a boolean evaluation basis $[T : F]$ where $p \in T$ and $q \in F$.

Here are three different grounds for $(p \lor \neg p) \lor \neg q$.

$$\dfrac{\dfrac{p \succ p}{p \succ p \lor \neg p}\,{\lor R_1}}{p \succ (p \lor \neg p) \lor \neg q}\,{\lor R_2}$$

Example

Take a boolean evaluation basis $[T : F]$ where $p \in T$ and $q \in F$.

Here are three different grounds for $(p \vee \neg p) \vee \neg q$.

$$\dfrac{\dfrac{p \succ p}{p \succ p \vee \neg p}\ \vee R_1}{p \succ (p \vee \neg p) \vee \neg q}\ \vee R_2 \qquad \dfrac{\dfrac{q \succ q}{\succ \neg q, q}\ \neg R}{\succ (p \vee \neg p) \vee \neg q, q}\ \vee R_1$$

Example

Take a boolean evaluation basis $[T : F]$ where $p \in T$ and $q \in F$.

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$$\dfrac{\dfrac{p \succ p}{\succ p \vee \neg p}\ \neg R}{\succ (p \vee \neg p) \vee q}\ \vee R_1$$

δ is a ground for A.

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δ is a derivation for $[\mathrm{L} : A, \mathrm{R}]$.

δ is a ground for A.

δ is a derivation for $[L : A, R]$.

δ is a ground against A.

δ is a ground for A.
δ is a derivation for $[L : A, R]$.

δ is a ground against A.
δ is a derivation for $[L, A : R]$.

2. Deduction

If δ is a derivation of $C \succ A, D$

and δ_1 is a ground for C,
and δ_2 is a ground against D

we can construct a ground for A like this:

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$$\dfrac{\dfrac{\begin{array}{c}\delta_1\\ \vdots\\ L \rhd C, R\end{array} \quad \begin{array}{c}\delta\\ \vdots\\ C \succ A, D\end{array}}{L \rhd A, D, R}\,\mathit{Cut} \qquad \begin{array}{c}\delta_2\\ \vdots\\ L, D \rhd R\end{array}}{L \rhd A, R}\,\mathit{Cut}$$

2. Deduction

If δ is a derivation of $C, A \succ D$

and δ_1 is a ground for C,
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we can construct a ground against A like this:

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we can construct a ground against A like this:

$$\dfrac{\dfrac{\begin{matrix}\delta_1\\ \vdots\\ L \rhd C, R\end{matrix} \qquad \begin{matrix}\delta\\ \vdots\\ C, A \succ D\end{matrix}}{L, A \rhd D, R}\,\mathit{Cut} \qquad \begin{matrix}\delta_2\\ \vdots\\ L, D \rhd R\end{matrix}}{L, A \rhd R}\,\mathit{Cut}$$

2. Deduction

If δ is a derivation of $C, A \succ D$

and δ_1 is a ground for C,

and δ_2 is a ground against D

we can construct a ground against A like this:

$$
\cfrac{\cfrac{\begin{array}{c}\delta_1\\ \vdots\\ L \rhd C, R\end{array} \qquad \begin{array}{c}\delta\\ \vdots\\ C, A \succ D\end{array}}{L, A \rhd D, R}\;\mathit{Cut} \qquad \begin{array}{c}\delta_2\\ \vdots\\ L, D \rhd R\end{array}}{L, A \rhd R}\;\mathit{Cut}
$$

(*This generalises*: a derivation of $X \succ A, Y$ [of $X, A \succ Y$] can be used to convert grounds for each member of X and against each member of Y into grounds for A [against A].)

We can think of a boolean valuation basis $[L : R]$
as a description of a *world*,
for a model of metaphysical grounding.

We can think of a boolean valuation basis $[L : R]$
as a description of a *world*,
for a model of metaphysical grounding.

We can think of a limited basis $[L : R]$
as modelling an *evidence base*,
for a model of epistemic grounding.

A ground is a derivation.
It is finite, and so
it can be grasped
(at least, in principle).

A ground for A is a derivation for $\mathrm{L} \rhd A, \mathrm{R}$.

It is not a ground for any other formula outside the basis $[\mathrm{L} : \mathrm{R}]$.

5. Hyperintensionality

A ground for A is a derivation for $\mathsf{L} \rhd A, \mathsf{R}$.

It is not a ground for any other formula outside the basis $[\mathsf{L} : \mathsf{R}]$.

Each valid formula has a ground (its derivation).

A ground for A is a derivation for $L \rhd A, R$.

It is not a ground for any other formula outside the basis $[L : R]$.

Each valid formula has a ground (its derivation).

If δ is a ground for A, and A entails B,
then we can ground B using δ
cut with the derivation of B.

6. Structure

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$$\dfrac{\dfrac{\begin{array}{c}\delta\\ \vdots\\ L \rhd A \to B, R\end{array} \qquad \dfrac{A \succ A \qquad B \succ B}{A, A \to B \succ B}\,{\to}L}{L, A \rhd B, R}\,\mathit{Cut} \qquad \begin{array}{c}\delta'\\ \vdots\\ L \rhd A, R\end{array}}{L \rhd B, R}\,\mathit{Cut}$$

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$$\dfrac{\begin{array}{c}\delta\\ \vdots\\ L \rhd A, R\end{array} \qquad \begin{array}{c}\delta'\\ \vdots\\ L \rhd B, R\end{array}}{L \rhd A \wedge B, R}\ {\scriptstyle \wedge R}$$

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$$\dfrac{\begin{array}{c}\delta\\ \vdots\\ L \rhd A, R\end{array} \qquad \begin{array}{c}\delta'\\ \vdots\\ L \rhd B, R\end{array}}{L \rhd A \wedge B, R}\ {\scriptstyle \wedge R}$$

$$\dfrac{\begin{array}{c}\delta\\ \vdots\\ L \rhd A \wedge B, R\end{array} \qquad \dfrac{A, B \succ A}{A \wedge B \succ A}\ {\scriptstyle \wedge R}}{L \rhd A, R}\ {\scriptstyle Cut} \qquad \dfrac{\begin{array}{c}\delta\\ \vdots\\ L \rhd A \wedge B, R\end{array} \qquad \dfrac{A, B \succ B}{A \wedge B \succ B}\ {\scriptstyle \wedge R}}{L \rhd B, R}\ {\scriptstyle Cut}$$

6. Structure

- A ground for $A \wedge B$ *can be seen consisting of* a ground for A and a ground for B.

$$\dfrac{\begin{array}{c}\delta\\ \vdots\\ L \rhd A, R\end{array} \qquad \begin{array}{c}\delta'\\ \vdots\\ L \rhd B, R\end{array}}{L \rhd A \wedge B, R}\ {\wedge R}$$

$$\dfrac{\begin{array}{c}\delta\\ \vdots\\ L \rhd A \wedge B, R\end{array} \qquad \dfrac{A, B \succ A}{A \wedge B \succ A}\ {\wedge R}}{L \rhd A, R}\ \mathit{Cut} \qquad \dfrac{\begin{array}{c}\delta\\ \vdots\\ L \rhd A \wedge B, R\end{array} \qquad \dfrac{A, B \succ B}{A \wedge B \succ B}\ {\wedge R}}{L \rhd B, R}\ \mathit{Cut}$$

Note: pairing grounds for A and grounds for B into a ground for $A \wedge B$, and then extracting a ground for A doesn't return the original ground for A, but a different ground.

Model 2

Model 1 satisfies our desiderata,
but there are many *many* different grounds
which do no epistemic or metaphysical *work*.

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$$
\dfrac{\dfrac{p \succ p \qquad \dfrac{p \succ p, A}{p \succ p \vee A}\,{\vee R}}{p, p \succ p \wedge (p \vee A)}\,{\wedge R}}{p \succ p \wedge (p \vee A)}\,W \qquad \dfrac{p \vee A, p \succ p}{(p \vee A) \wedge p \succ p}\,{\wedge L}
$$
$$
\overline{\qquad\qquad\qquad\qquad p \succ p \qquad\qquad\qquad\qquad}\;\mathit{Cut}
$$

(There's a different one of these for each formula A.)

Model 2

$$
\dfrac{\dfrac{\dfrac{\dfrac{p \succ p}{p, q \succ p, q}\,K}{p \wedge q \succ p, q}\,\wedge L}{p \wedge q \succ p \vee q}\,\vee R}{\succ (p \wedge q) \rightarrow (p \vee q)}\,\rightarrow R
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{p \succ p}{p, q \succ p, q}\,K}{p, q \succ p \vee q}\,\vee R}{p \wedge q \succ p \vee q}\,\wedge L}{\succ (p \wedge q) \rightarrow (p \vee q)}\,\rightarrow R
$$

(These derivations differ in ways that don't matter for how $(p \wedge q) \rightarrow (p \vee q)$ is grounded.)

Model 2

$$\dfrac{\dfrac{\dfrac{\dfrac{p \succ p}{p, q \succ p, q}\,K}{p \wedge q \succ p, q}\,{\wedge}L}{p \wedge q \succ p \vee q}\,{\vee}R}{\succ (p \wedge q) \to (p \vee q)}\,{\to}R
\qquad\qquad
\dfrac{\dfrac{\dfrac{\dfrac{p \succ p}{p, q \succ p, q}\,K}{p, q \succ p \vee q}\,{\vee}R}{p \wedge q \succ p \vee q}\,{\wedge}L}{\succ (p \wedge q) \to (p \vee q)}\,{\to}R$$

(These derivations differ in ways that don't matter for how $(p \wedge q) \to (p \vee q)$ is grounded. They have the same *proof term*.)

Model 2

$$\dfrac{\dfrac{\dfrac{\dfrac{p \succ p}{p, q \succ p, q}\,K}{p \wedge q \succ p, q}\,{\wedge}L}{p \wedge q \succ p \vee q}\,{\vee}R}{\succ (p \wedge q) \to (p \vee q)}\,{\to}R \qquad \dfrac{\dfrac{\dfrac{\dfrac{p \succ p}{p, q \succ p, q}\,K}{p, q \succ p \vee q}\,{\vee}R}{p \wedge q \succ p \vee q}\,{\wedge}L}{\succ (p \wedge q) \to (p \vee q)}\,{\to}R$$

(These derivations differ in ways that don't matter
for how $(p \wedge q) \to (p \vee q)$ is grounded.
They have the same *proof term*.)

Given a basis $[L : R]$,

a ground for A is a proof term
of a derivation for $L \rhd A, R$.

and a ground against A is a proof term
of a derivation for $L, A \rhd R$.

Example

Here are the grounds for $p \vee \neg p$
in any basis including $[p : \ \ ]$.

$$p \succ p \vee \neg p$$

$$p \succ p \vee \neg p$$

$$p \succ p \vee \neg p$$

There are many fewer *proof term* grounds.

Each different ground represents a different way for formula to be grounded in the basis.

Beyond

- Proof term grounds allow for only one ground for (or against) each atomic formula. Is this appropriate? How can this restriction best be lifted?
- Are there other models of this general shape?
- What is an appropriate basis for a metaphysical (total) interpretation for grounds for sentences in predicate logic?

THANK YOU!

Thank you!

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or *email* at `restall@unimelb.edu.au`