THE SEMANTICS & PSYCHOLOGY OF NEGATION:

THE AUSTRALIAN PLAN, NEGATION AS FAILURE, AND CARD SELECTION TASKS

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ARCHÉ M&L SEMINAR ☆ 20 SEPTEMBER 2023

* THIS TALK IS BASED ON JOINT WORK WITH FRANCESCO BERTO

https://consequently.org/presentation

MY PLAN

1. SCENE SETTING
2. TRUTH CONDITIONS FOR NEGATION
3. TAKING TWO DIFFERENT PERSPECTIVES
4. CARD SELECTION TASKS
5. WHERE TO GO FROM HERE

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This is joint work with
my colleague Francesco Berto.

Journal of Philosophical Logic
https://doi.org/10.1007/s10992-019-09510-2

Negation on the Australian Plan

Francesco Berto[1,2] · **Greg Restall**[3]

Received: 25 November 2017 / Accepted: 30 March 2019 / Published online: 22 April 2019

Abstract

We present and defend the Australian Plan semantics for negation. This is a comprehensive account, suitable for a variety of different logics. It is based on two ideas. The first is that negation is an exclusion-expressing device: we utter negations to express incompatibilities. The second is that, because incompat*ibility* is modal, negation is a modal operator as well. It can, then, be modelled as a quantifier over points in frames, restricted by accessibility relations representing compatibilities and incompatibilities between such points. We defuse a number of objections to this Plan, raised by supporters of the American Plan for negation, in which negation is handled via a many-valued semantics. We show that the Australian Plan has substantial advantages over the American Plan.

Keywords Negation · Compatibility semantics · Kripke semantics · Non-classical logics · Many-valued logics · Modal logics

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We are interested in the semantics of logical vocabulary, and how this connects with what we do in our thought and talk.

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We are interested in the semantics of logical vocabulary, and how this connects with what we do in our thought and talk.

Today I'll talk about the semantics of negation and some connections with the psychology of reasoning.

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$\neg A$ is true if and only if A is not true.

In terms of situations

$s \Vdash \neg A$ if and only if $s \nVdash A$

In terms of truth values

$v(\neg A) = 1$ if and only if $v(A) \neq 1$

(i.e., when $v(A) = 0$)

Generalising Truth Values

$v(\neg A) = 1$ if and only if $v(A) = 0$

$v(\neg A) = 0$ if and only if $v(A) = 1$

Generalising Truth Values

$v(\neg A) = 1$ if and only if $v(A) = 0$

$v(\neg A) = i$ if and only if $v(A) = i$

$v(\neg A) = 0$ if and only if $v(A) = 1$

If the intermediate value is taken to be neither true nor false, we have a truth-value gap.

Generalising Truth Values

$v(\neg A) = 1$ if and only if $v(A) = 0$

$v(\neg A) = i$ if and only if $v(A) = i$

$v(\neg A) = 0$ if and only if $v(A) = 1$

If the intermediate value is taken to be both true and false, we have a truth-value glut.

Generalising Truth Values

$v(\neg A) = 1$ if and only if $v(A) = 0$

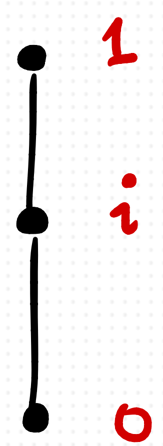
$v(\neg A) = n$ if and only if $v(A) = n$

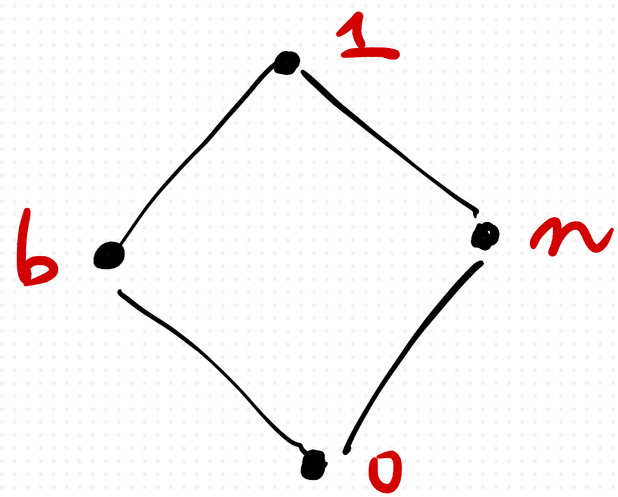
$v(\neg A) = b$ if and only if $v(A) = b$

$v(\neg A) = 0$ if and only if $v(A) = 1$

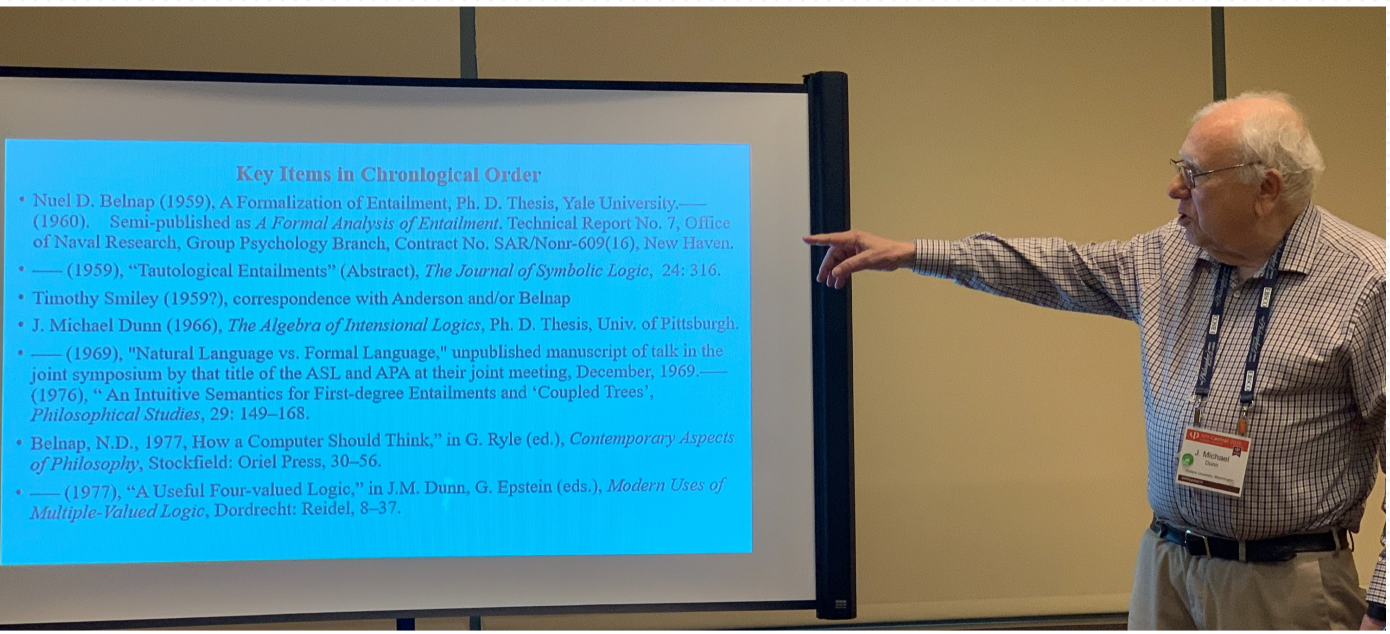
If you really want, you can have two intermediate values for 'both' and 'neither' — gluts & gaps.

(If you wonder how to evaluate the other logical operators in schemes like this, meditate on these Hasse diagrams. Conjunction is greatest lower bound, disjunction, least upper bound, as usual.)

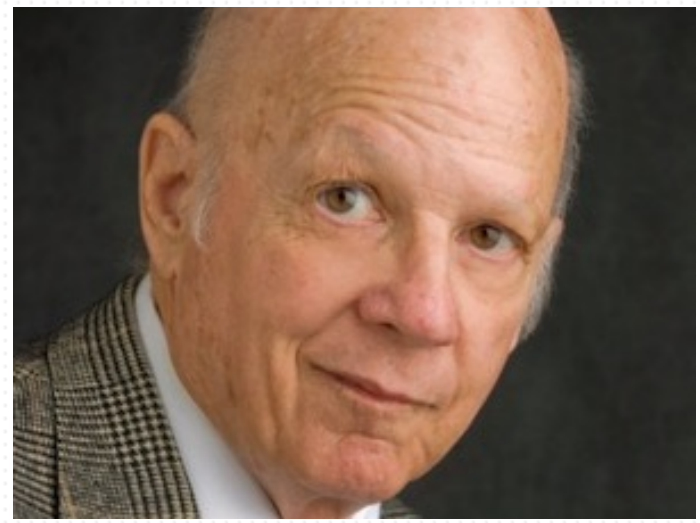




In the relevant logic tradition, this scheme for negation (generalising beyond two truth values) is called the **AMERICAN PLAN**, because it* comes from the work of the two American logicians

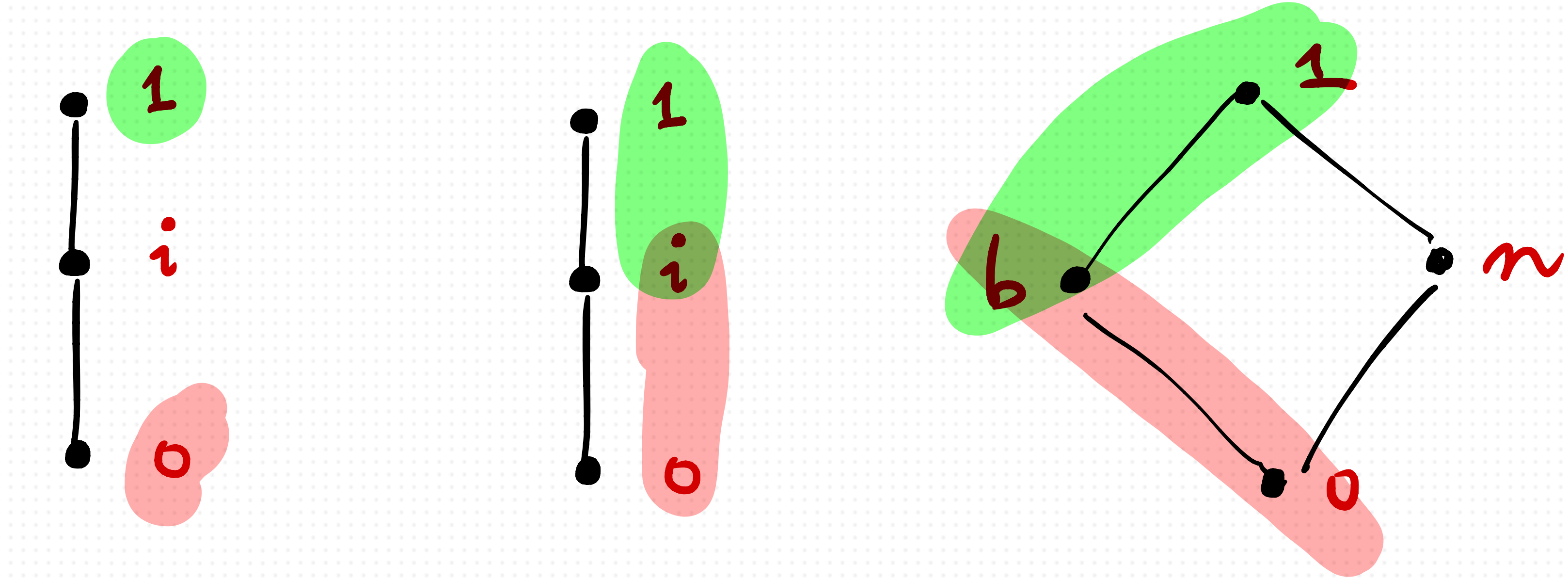


J. MICHAEL DUNN



NUEL BELNAP

* In this tradition, at least. The idea arose elsewhere, too.



The distinctive feature of these semantic schemes is that truth and falsity are treated on a par as distinct (though connected) semantic statuses.

There are other ways to generalise Boolean negation.

$S \Vdash \neg A$ if and only if $S \nVdash A$.

Beth/Kripke Semantics for Intuitionistic Logic

$s \Vdash \neg A$ iff for every $t \geq s$, $t \nVdash A$.

The Routley Star Semantics

$s \Vdash \neg A$ if and only if $s^* \nVdash A$.

The General Scheme....

$s \Vdash \neg A$ iff for every t where sCt, $t \nVdash A$.

This scheme, in which negation is given a truth-conditional semantics by way of a context-shift 'compatibility' relation has become known as the AUSTRALIAN plan, because it arose* in the work of Australian logicians

Valerie Plumwood
(then Routley)

Richard Sylvan
(then Routley)

* In this tradition, at least. The idea arose elsewhere, too.

The distinctive feature of these semantic schemes is that truth and falsity are treated differently. Falsity (truth of a negation), arises out of truth & (in)compatibility.

These two plans are very different, and some take them to be in conflict.

J Philos Logic
DOI 10.1007/s10992-017-9427-0

There is More to Negation than Modality

Michael De[1] · Hitoshi Omori[2]

Received: 17 February 2016 / Accepted: 20 January 2017

Abstract There is a relatively recent trend in treating negation as a modal operator. One such reason is that doing so provides a uniform semantics for the negations of a wide variety of logics and arguably speaks to a longstanding challenge of Quine put to non-classical logics. One might be tempted to draw the conclusion that negation is a modal operator, a claim Francesco Berto (*Mind, 124*(495), 761–793, 2015) defends at length in a recent paper. According to one such modal account, the negation of a sentence is true at a world x just in case all the worlds at which the sentence is true are *incompatible* with x. Incompatibility is taken to be the key notion in the account, and what minimal properties a negation has comes down to which minimal conditions incompatibility satisfies. Our aims in this paper are twofold. First, we wish to point out problems for the modal account that make us question its tenability on a fundamental level. Second, in its place we propose an alternative, non-modal, account of negation as a contradictory-forming operator that we argue is superior to, and more natural than, the modal account.

Keywords Negation · Compatibility · Modality · Contradictory

This research was funded by the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013)/ERC Grant agreement nr 263227. Hitoshi Omori is a Postdoctoral Research Fellow of the Japan Society for the Promotion of Science (JSPS).

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My task here is not to ajudicate this dispute, but to explore one of the ways the distinctive features of the Australian Plan semantics can be applied.

Before that, let's see another tradition in the semantics of negation: NEGATION AS FAILURE, from logic programming & database theory.

NEGATION AS FAILURE

Keith L. Clark

Department of Computer Science & Statistics

Queen Mary College, London, England

ABSTRACT

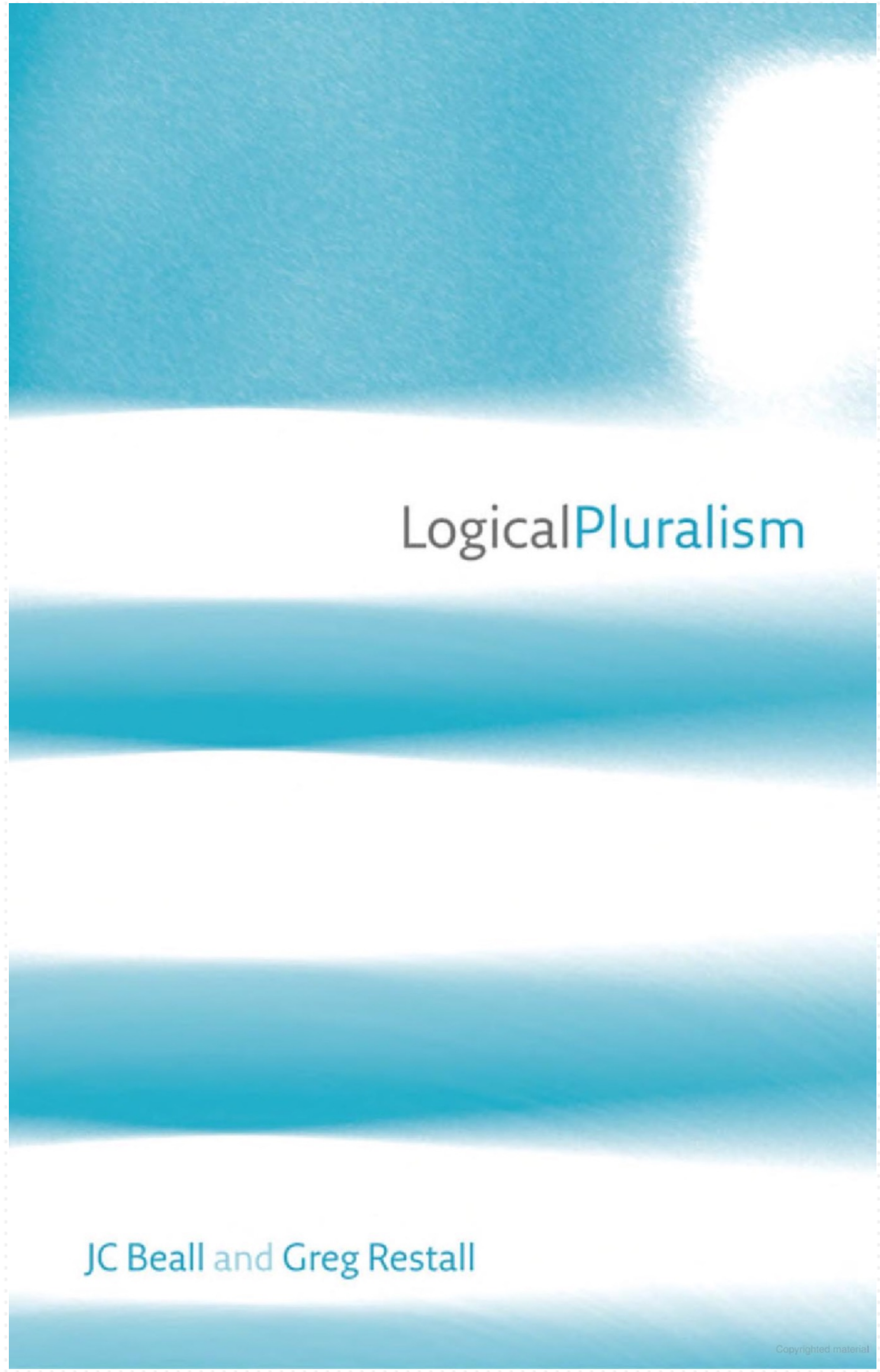
A query evaluation process for a logic data base comprising a set of clauses is described. It is essentially a Horn clause theorem prover augmented with a special inference rule for dealing with negation. This is the <u>negation as failure</u> inference rule whereby ~ P can be inferred if every possible proof of P <u>fails</u>. The chief advantage of the query evaluator described is the effeciency with which it can be implemented. Moreover, we show that the negation as failure rule only allows us to conclude negated facts that could be inferred from the axioms of the completed data base, a data base of relation definitions and equality schemas that we consider is implicitly given by the data base of clauses. We also show that when the clause data base and the queries satisfy certain con-

Treat a database D as verifying $\neg A$ if and only if D fails to verify A.

(This looks a lot like Boolean Negation, but this is a database, not a world.)

Which of these approaches is CORRECT?

I am not the person to give you a direct answer to that kind of question.



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Why don't we have both?

I will propose a view from which both NEGATION AS FAILURE and an AUSTRALIAN PLAN semantics for negation can explain different aspects of the psychology of reasoning with negations.

THE FRAMEWORK

S H A

THE FRAMEWORK

$$S \Vdash A$$

An agent's evidence base

A judgement

THE FRAMEWORK

$$s \Vdash A$$

An agent's evidence base

A judgement

Evidence bases are not worlds.

Judgements are not sets of worlds.

THE FRAMEWORK

$$S \Vdash A$$

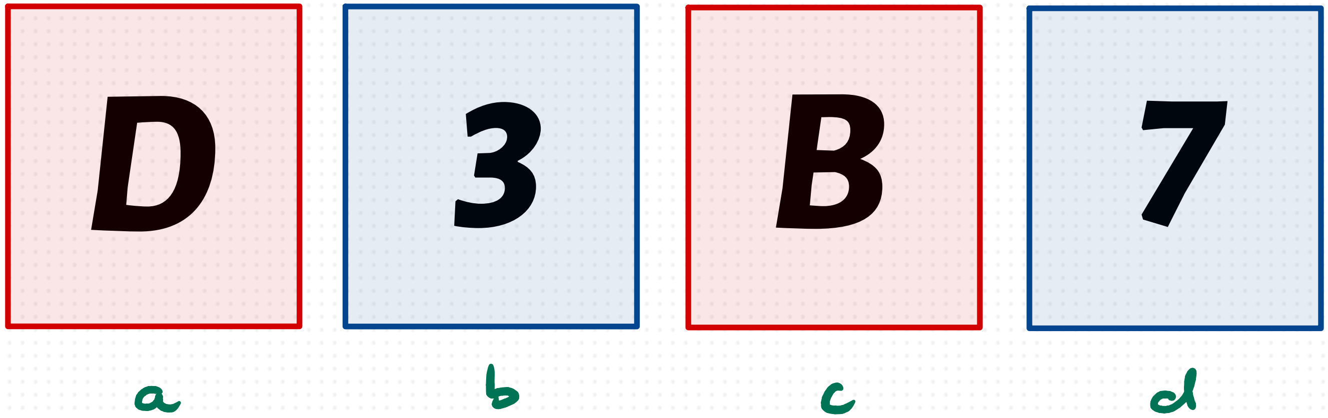
ACCORDING TO S, A holds (or is given).

An agent's evidence base

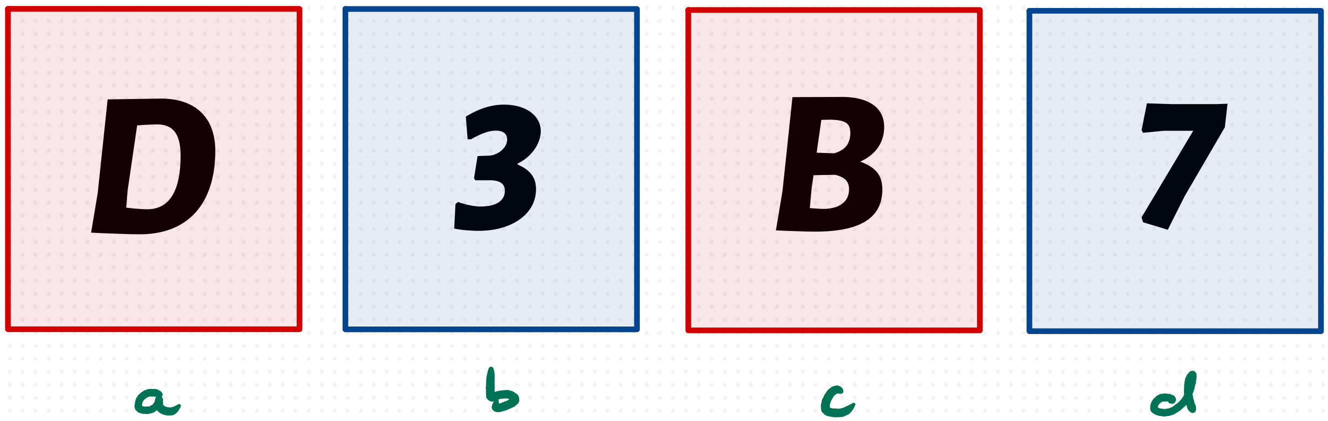
A judgement

Evidence bases are not worlds.

Judgements are not sets of worlds.

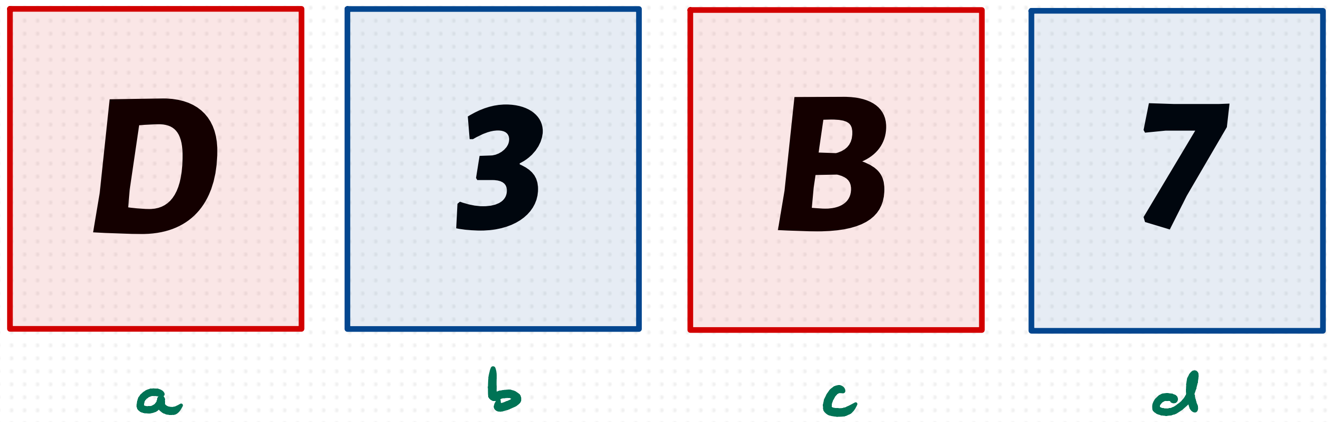


S	D	B	3	7
a	✓	–	–	–
b	–	–	✓	–
c	–	✓	–	–
d	–	–	–	✓



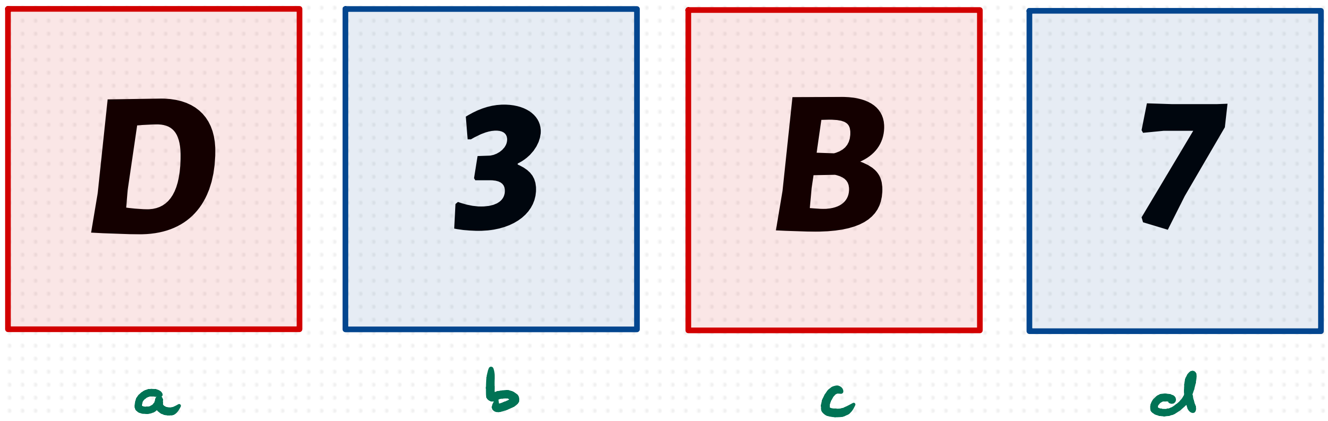
s	D	B	3	7
a	✓	–	–	–
b	–	–	✓	–
c	–	✓	–	–
d	–	–	–	✓

$s \Vdash Da \qquad s \Vdash 3b$

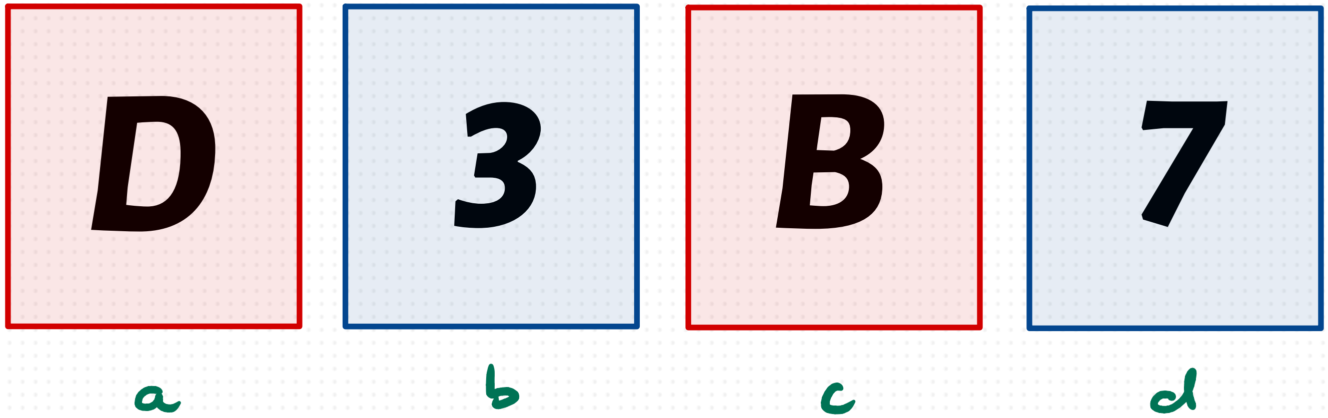


S	D	B	3	7
a	✓	–	–	–
b	–	–	✓	–
c	–	✓	–	–
d	–	–	–	✓

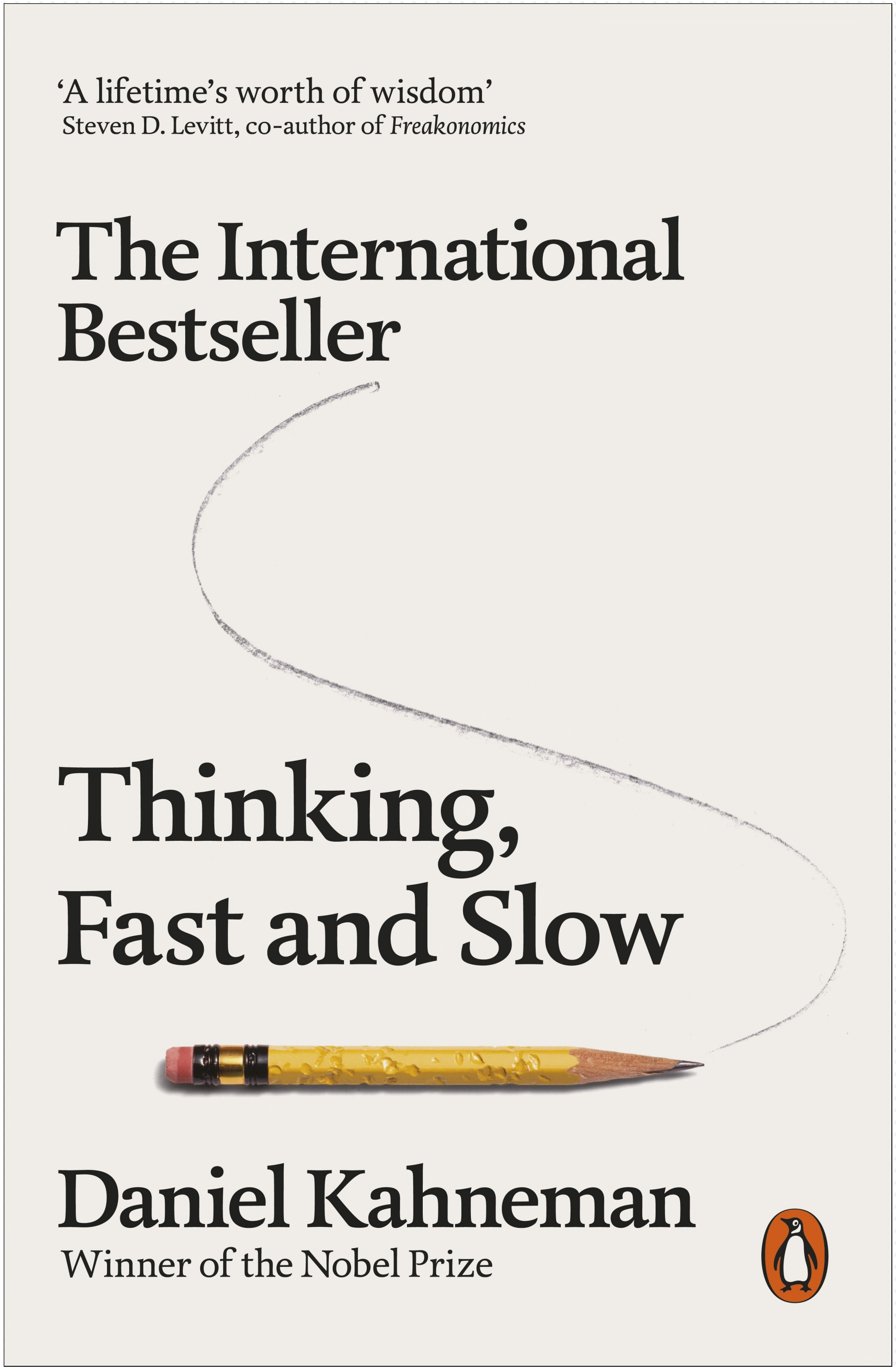
$s \Vdash Da$ $\quad s \Vdash 3b$ $\quad s \nVdash Db$ $\quad s \nVdash 7b$



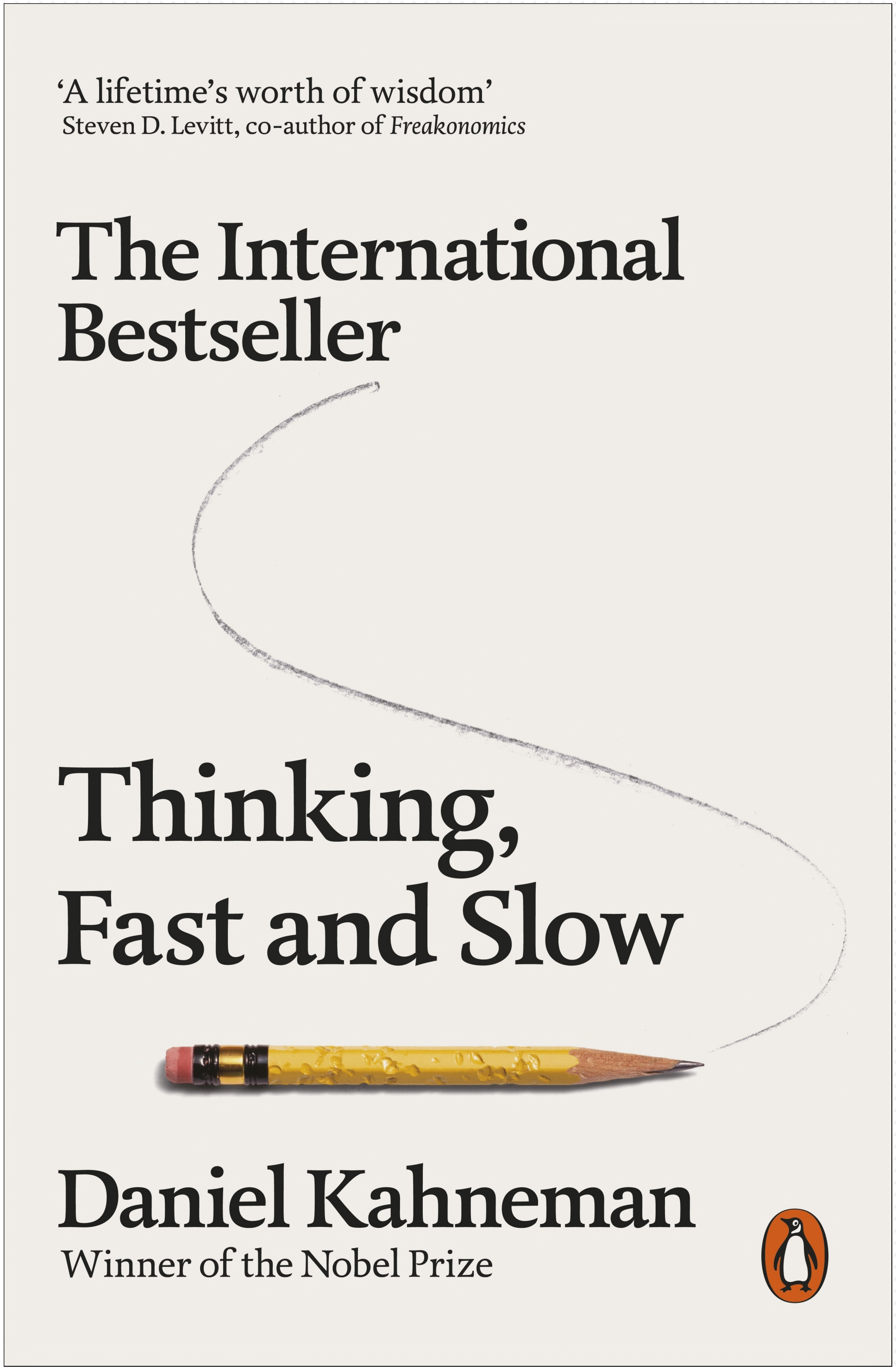
When does this evidential situation support a negative judgement, like $\neg 7b$ or $\neg Db$?



Well, it depends on what you mean.



I'll take for granted that there are different kinds of cognitive processes involved in our information processing, including in our treatment of negation & negative judgements.



Let's work with two levels of information processing

$S \Vdash_1 A$ — fast SYSTEM 1

$S \Vdash_2 A$ — slow SYSTEM 2

because we're interested in the psychology of reasoning.

Immediate, fast reaction judgement

$S \Vdash_1 A$ (A a basic judgement) iff $S \Vdash A$

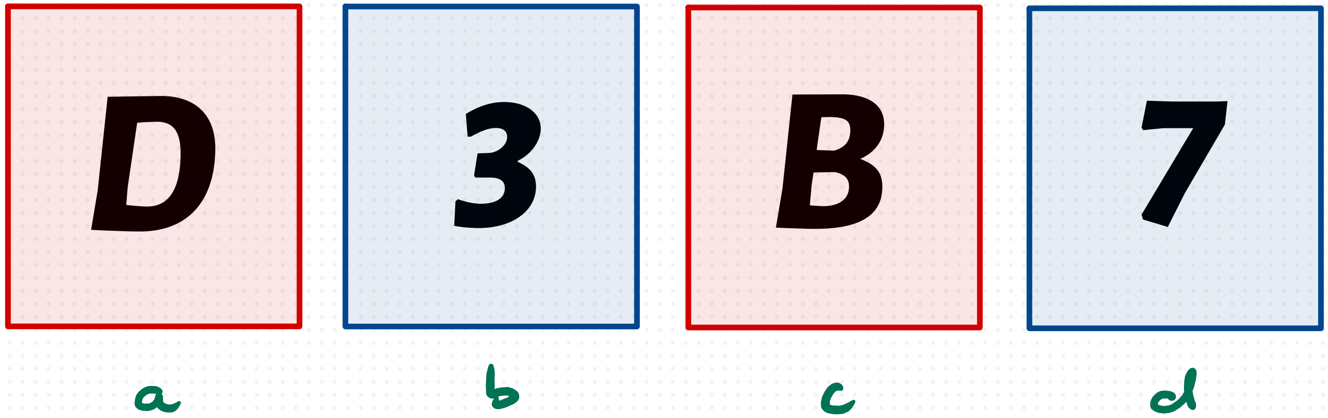
$S \Vdash_1 \neg A$ if and only if $S \nVdash_1 A$

Immediate, fast reaction judgement

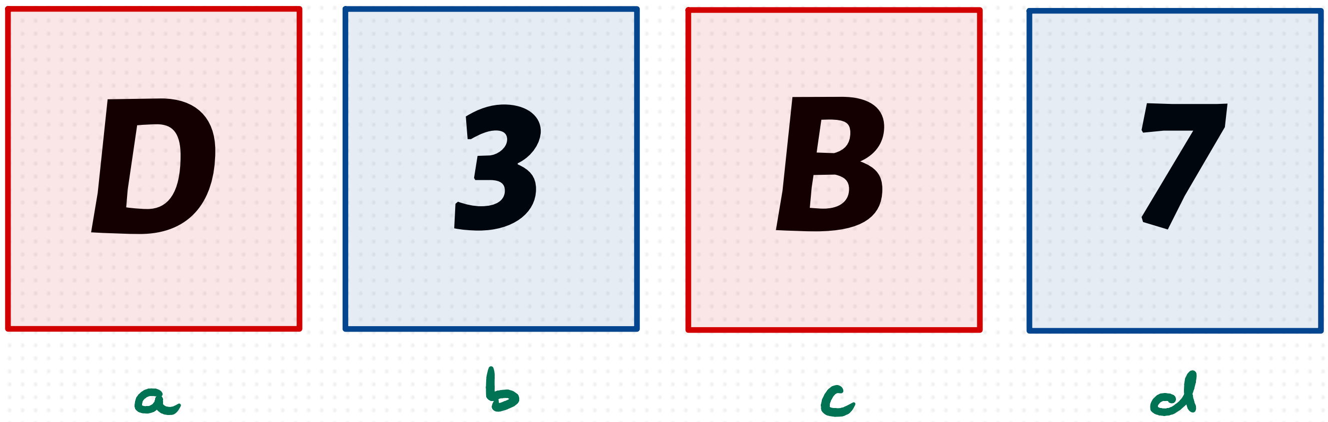
$S \Vdash_1 A$ (A a basic judgement) iff $S \Vdash A$

$S \Vdash_1 \neg A$ if and only if $S \nVdash_1 A$

(At least when A is a basic judgement.
I leave it an open question whether
System 1 can deliver claims such as $\neg\neg Da$)

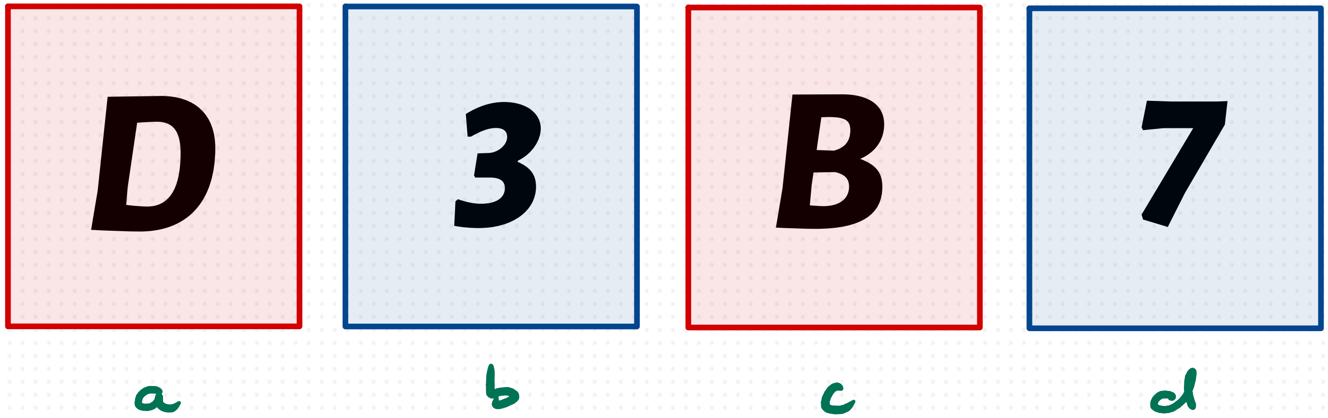


$S \Vdash_1 Da \qquad S \Vdash_1 \neg 3a \qquad S \Vdash_1 \neg Ba \qquad S \Vdash_1 \neg 7a$



$s \Vdash_1 Da \qquad s \Vdash_1 \neg 3a \qquad s \Vdash_1 \neg Ba \qquad s \Vdash_1 \neg 7a$

$s \Vdash_1 \neg Db \qquad s \Vdash_1 3b \qquad s \Vdash_1 \neg Bb \qquad s \Vdash_1 \neg 7b$



$S \Vdash Da \qquad S \Vdash \neg 3a \qquad S \Vdash \neg Ba \qquad S \Vdash \neg 7a$

But clearly, these are not all alike,
if you know about the card setup
& you think for a little bit.

D	3	B	7
a	b	c	d

$S \Vdash Da \qquad S \nVdash \neg 3a \qquad S \Vdash \neg Ba \qquad S \Vdash \neg 7a$

Card a doesn't have a 3 on this side, but it <u>might</u> on the other.

D	3	B	7
a	b	c	d

$S \Vdash Da$ $\quad S \nVdash \neg 3a$ $\quad S \Vdash \neg Ba$ $\quad S \nVdash \neg 7a$

Card a doesn't have a 3 on this side, but it *might* on the other.

If the evidence base contains the constraint that each card has a letter on one side & a number on the other...
Some reasoning can deliver this negative judgement.

D	3	B	7
a	b	c	d

$s \Vdash_2 Da$

$s \nVdash_2 \neg 3a \qquad s \Vdash_2 \neg Ba \qquad s \nVdash_2 \neg 7a$

We think these sorts of distinctions take a bit more work to make. They seem more like slow thinking: System 2.

System 2, slow reaction judgement

$s \Vdash_2 A$ (A a basic judgement) iff $s \Vdash A$

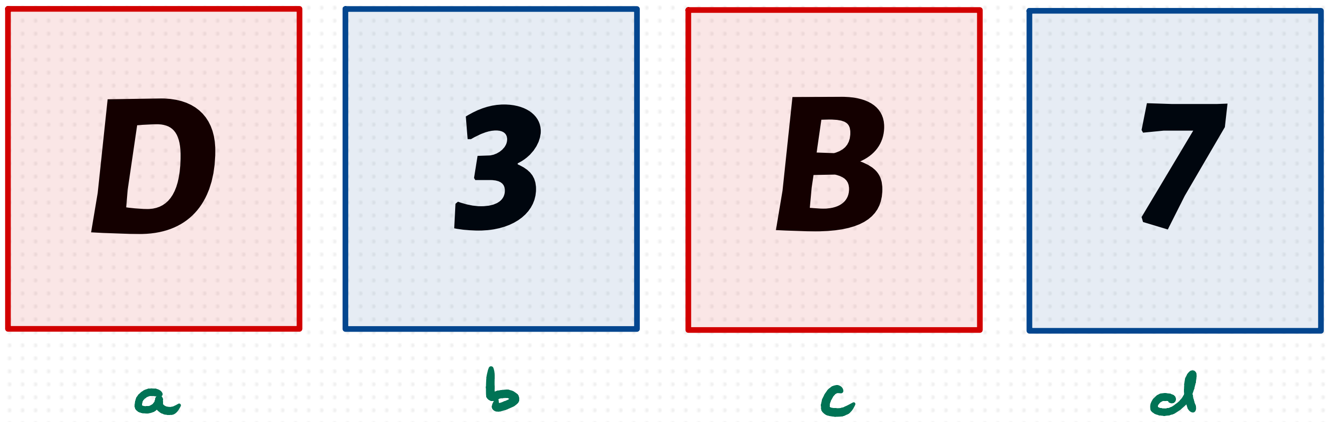
$s \Vdash_2 \neg A$ if and only if $t \nVdash_2 A$, for any t compatible with s.

System 2, slow reaction judgement

$s \Vdash_2 A$ (A a basic judgement) iff $s \Vdash A$

$s \Vdash_2 \neg A$ if and only if $t \nVdash_2 A$, for any t compatible with s.

This requires each evidence base to not only support basic judgements, but a compatibility relation between evidence bases — and system 2 reflection must operate on those hypothetical evidence bases!



s	D	B	3	7
a	✓	–	–	–
b	–	–	✓	–
c	–	✓	–	–
d	–	–	–	✓

t	D	B	3	7
a	✓	–	✓	–
b	–	–	✓	–
c	–	✓	–	–
d	–	–	–	✓

a could have a 3 on the other side

u	D	B	3	7
a	✓	–	–	✓
b	–	–	✓	–
c	–	✓	–	–
d	–	–	–	✓

or it might be a 7.

This sort of considered reflection of alternatives seems to model the way we reason about negations when we take our time.

Fast, easy, System 1 NEGATION AS FAILURE
(overgenerates)

This sort of considered reflection of alternatives seems to model the way we reason about negations when we take our time.

Slow, difficult, system 2 AUSTRALIAN PLAN
compatibility negation (accurate)

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D	3	B	7
a	b	c	d

Every card has a letter on one side & a number on the other

Which cards must you flip to verify "If a card has a D on one side there is a 3 on the other"?

REASONING ABOUT A RULE

BY

P. C. WASON

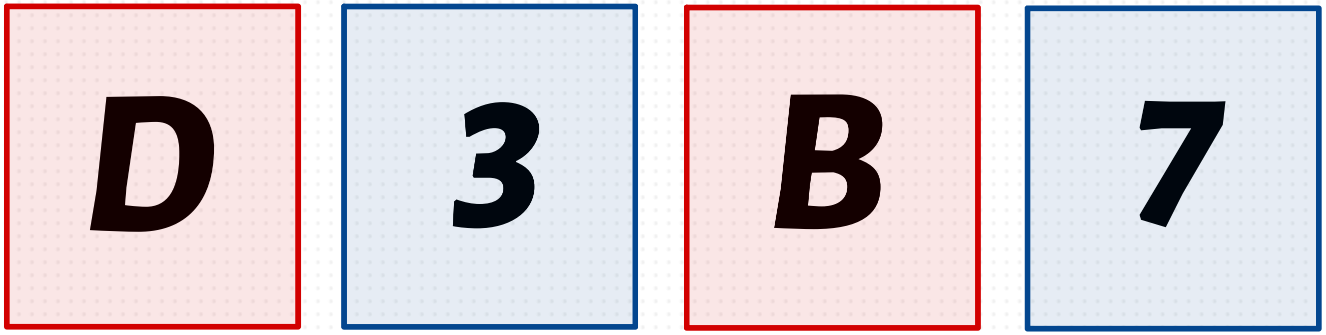
From Psycholinguistics Research Unit, University College London

Two experiments were carried out to investigate the difficulty of making the contrapositive inference from conditional sentences of the form, "if P then Q." This inference, that not-P follows from not-Q, requires the transformation of the information presented in the conditional sentence. It is suggested that the difficulty is due to a mental set for expecting a relation of truth, correspondence, or match to hold between sentences and states of affairs. The elicitation of the inference was not facilitated by attempting to induce two kinds of therapy designed to break this set. It is argued that the subjects did not give evidence of having acquired the characteristics of Piaget's "formal operational thought."

Quarterly J. Exp. Psych. 1968

Introduction

This investigation is concerned with the difficulty of making a particular type



a b c d

Every card has a letter on one side & a number on the other

Which cards must you flip to verify "If a card has a D on one side there is a 3 on the other"?

Fewer than 10% of the participants answered correctly (a & d).

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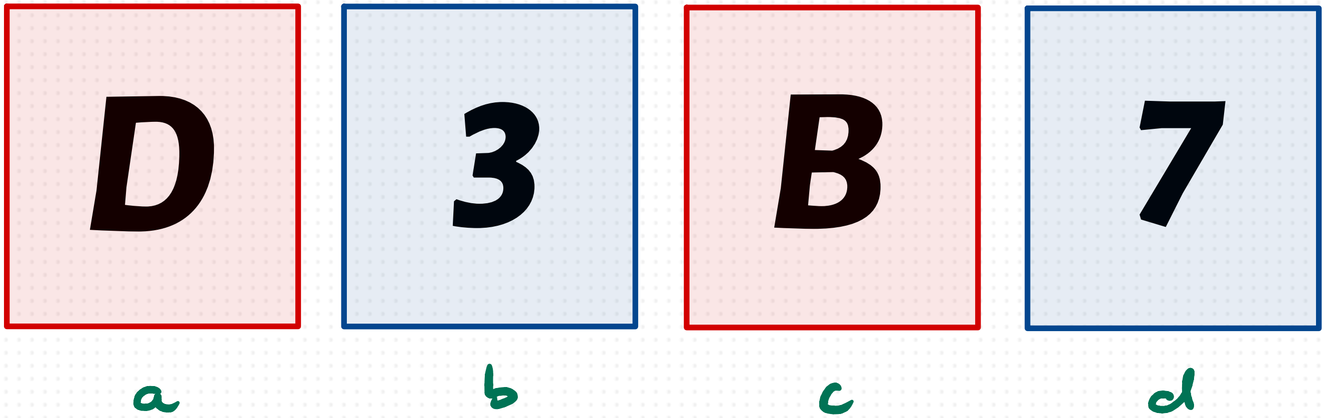

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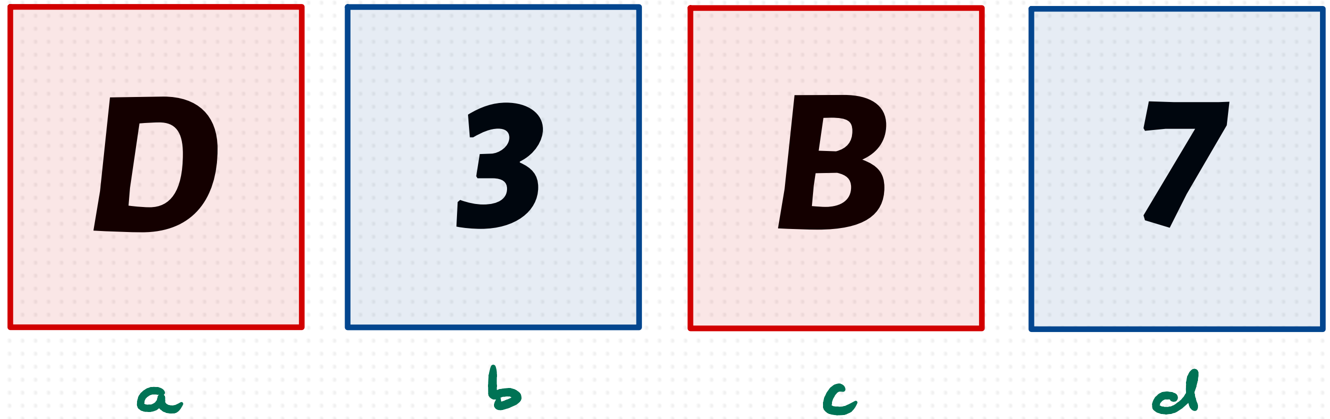

Perhaps surprisingly, performance is much better if you negate the consequent. "If a card has a D on one side there isn't a 3 on the other." (Choose a & b.)

Br. J. Psychol. (1973), **64**, 3, *pp.* 391–397
Printed in Great Britain

MATCHING BIAS IN THE SELECTION TASK

By J. St B. T. EVANS and J. S. LYNCH
Psychology Section, City of London Polytechnic

A previous study (Evans, 1972) found that subjects tend to match rather than alter named values when constructing verifying and falsifying cases of conditional rules. It was suggested that this tendency ('matching bias') might account for the responses normally observed in Wason's (1968, 1969) 'selection task'. This suggestion was tested by giving subjects the selection task with conditional rules in which the presence and absence of negative components was systematically varied, to see whether subjects consistently attempted to verify the rules (Wason's theory) or whether they continued to choose the matching values despite the presence of negatives, which would reverse the logical meaning of such selections. Significant matching tendencies were observed on four independent measures, and the overall pattern, with matching bias cancelled out, gave no evidence for a verification bias, indicating instead that the logically correct values were most frequently chosen.

Wason & Johnson-Laird (1972) review a number of recent studies about the reasoning patterns generally obtained in Wason's 'selection task'. That task was

Reasoning about a **rule**

PC Wason - Quarterly journal of experimental psychology, 1968 - journals.sagepub.com

Two experiments were carried out to investigate the difficulty of making the contra-positive inference from conditional sentences of the form, “if P then Q.” This inference, that not-P …

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!!!

There is a vast literature on card selection tasks!

It is not our aim to get to the bottom of all of it.

We want to see how contemporary work in the semantics of negation can be tested for cognitive significance.

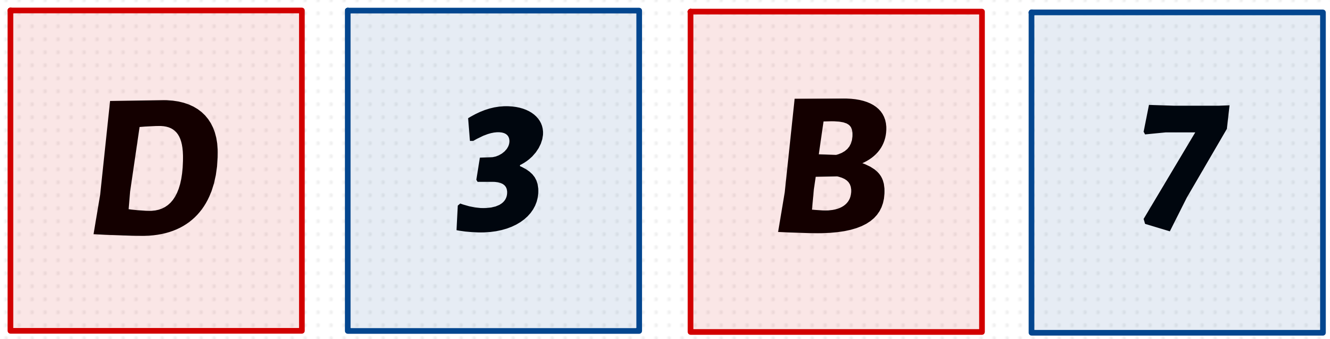
Insight 1: Reasoning accurately about negations (and falsity) involves generalising over compatible evidence bases, and this is complicated. It is not surprising that we find this difficult.

D	3	B	7
a	b	c	d

$S \Vdash_1 Da$ $\quad S \Vdash_1 \neg 3a$ $\quad S \Vdash_1 \neg Ba$ $\quad S \Vdash_1 \neg 7a$

$S \Vdash_1 \neg Db$ $\quad S \Vdash_1 3b$ $\quad S \Vdash_1 \neg Bb$ $\quad S \Vdash_1 \neg 7b$

Insight 2: If System 1 judgements about negations are quick-and-dirty negation as failure judgements, it's not surprising that we overgenerate answers.



a b c d

How can we account for greater success in the negated consequent form of the task:

"If there is a D on one side of the card there isn't a 3 on the other"?

Here we might use some concepts from Berto's 2022 book Topics of Thought.

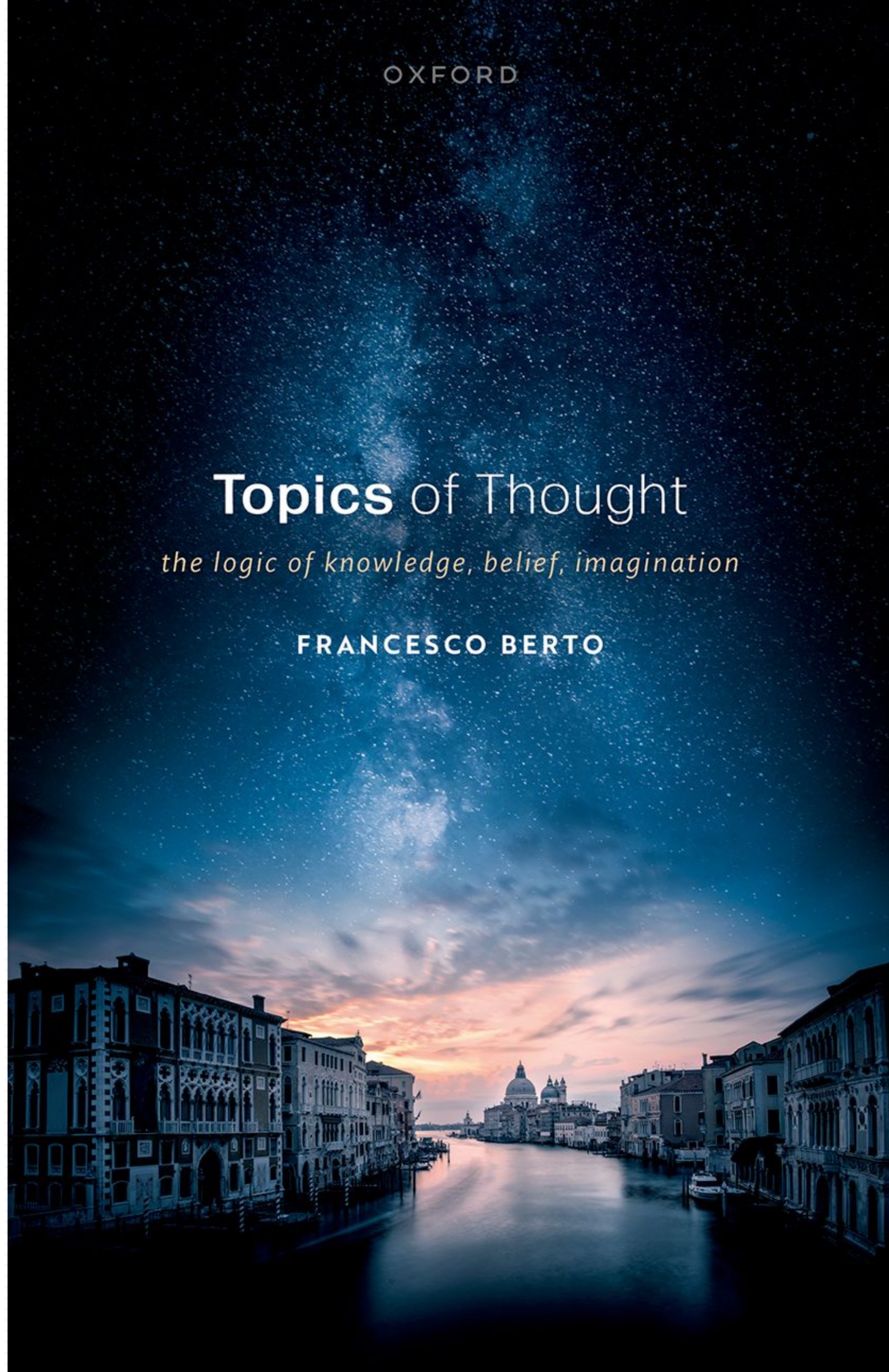
Judgements do not only have truth conditions — they also have topics.

Negation is topic-transparent.

$$t(\neg A) = t(A).$$

So is the material conditional.

$$t(A \to B) = t(A) \oplus t(B).$$

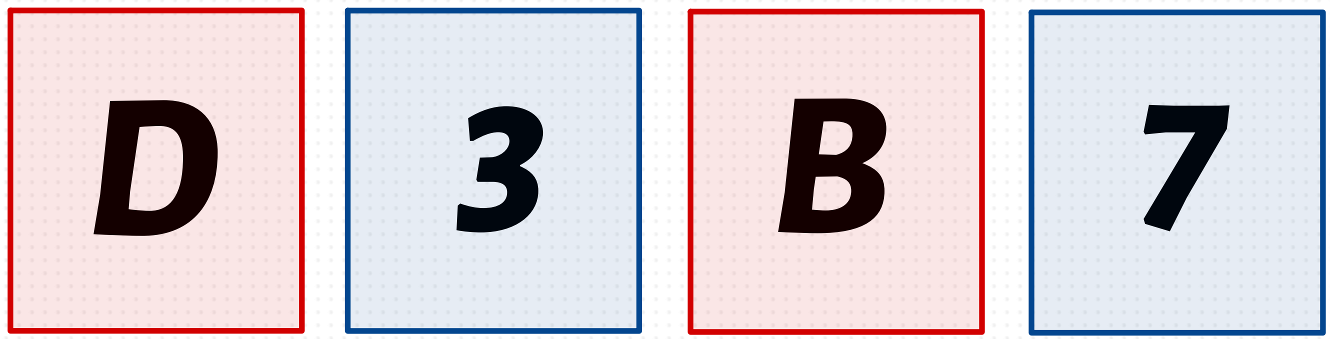


D	3	B	7
a	b	c	d

"If there is a D on one side of the card there isn't a 3 on the other"?

$$t(Dx \to \neg 3x) = t(Dx) \oplus t(3x)$$

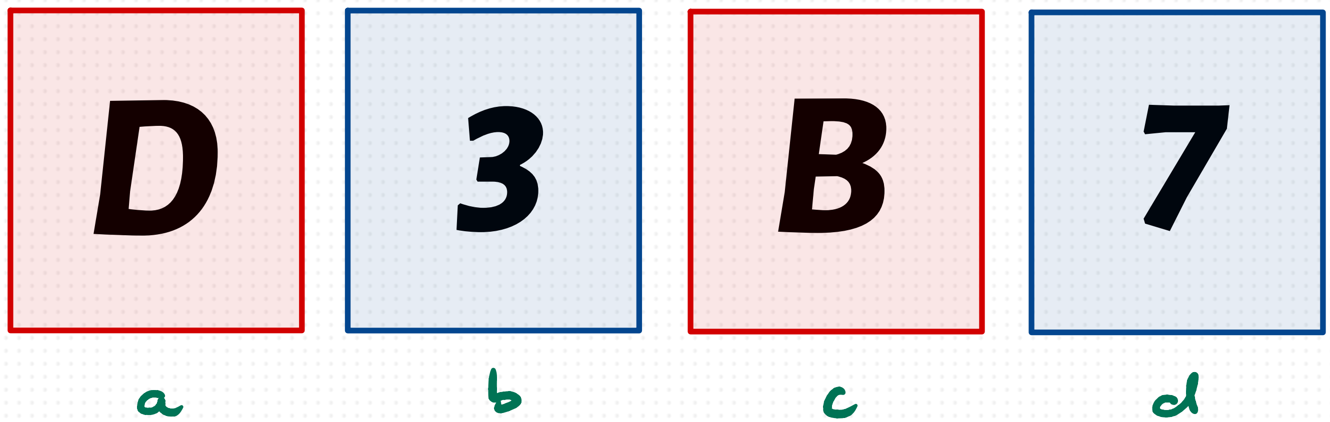
$$t(Dx \to 3x) = t(Dx) \oplus t(3x)$$



a b c d

"If there is a D on one side of the card there isn't a 3 on the other"?

If our pre-reflective quick judgement of relevance is guided by topic (in this sense) then it is not surprising that we might pick a & b (at least) in this scenario, whether we check $Dx \to \neg 3x$ or $Dx \to 3x$, since being a D & being a 3 is clearly on topic.



"If there is a D on one side of the card there isn't a 3 on the other"?

If we stop there, to consider only the clearly D and 3 cards, without considering the other sides of c & d, we chance on the right answer of the $Dx \to \neg 3x$ task, but err on the $Dx \to 3x$ task.

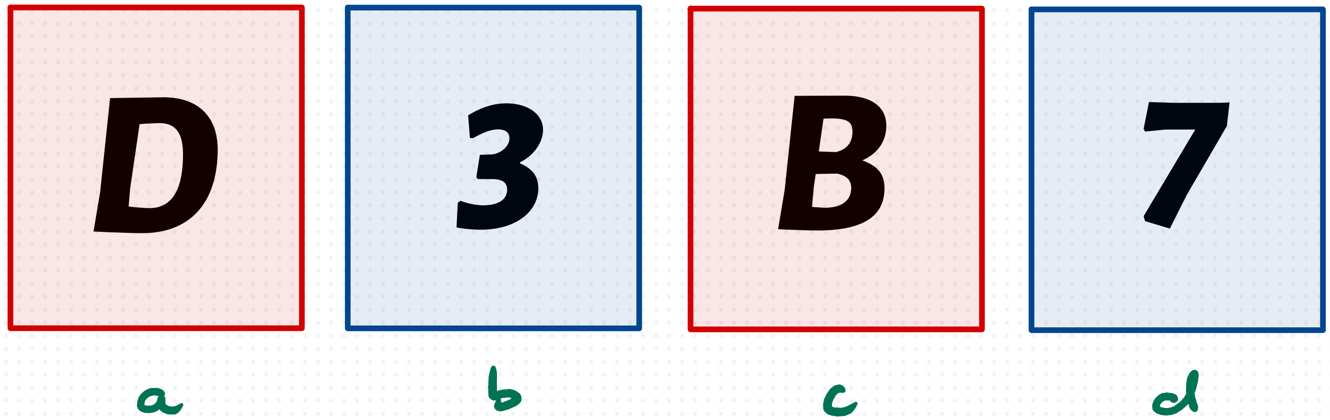
D	3	B	7
a	b	c	d

"If there is a D on one side of the card there isn't a 3 on the other"?

Combining topic sensitivity with negation as failure (System 1) judgements brings every card into salience, which could explain why people are prone to overgenerate answers in either case.

D	3	B	7
a	b	c	d

Contemporary work in the philosophy of logic can give us new ideas about possible cognitive mechanisms at play in our reasoning judgements, whether fast or slow.



Contemporary work in the philosophy of logic can give us new ideas about possible cognitive mechanisms at play in our reasoning judgements, whether fast or slow.

There does not need to be a one-size-fits-all approach. Pluralism seems fitting here!

MY PLAN

1. SCENE SETTING
2. TRUTH CONDITIONS FOR NEGATION
3. TAKING TWO DIFFERENT PERSPECTIVES
4. CARD SELECTION TASKS
5. WHERE TO GO FROM HERE?

This work is only just beginning!

1. Read through existing results with logically-informed eyes.
2. Examine the logical literature for cognitively significant tools.
3. Make conjectures, and test them.
4. Refine the conjectures & repeat...

Thanks!