What’s so Special about Logic?
Practices, Rules and Definitions

Greg Restall

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MELBOURNE

ALICE AMBROSE LAZEROWITZ – THOMAS TYMOCZKO MEMORIAL LOGIC LECTURE
4 DECEMBER 2019 / SMITH COLLEGE
To understand *logic* better...
To understand *logic* better...

... and especially, to understand what makes logic *distinctive*. 
A CONTROVERSY IN THE LOGIC OF MATHEMATICS

THE impetus of the attack on Cantor’s theory of transfinite carried assailants and defenders into an examination of the foundations of mathematics, and thence into a questioning of logic as a substructure for mathematics. Is logic basic, as Frege and Peano assumed, and are its laws final? Brouwer has answered this question with a flat no. Scorning any grounding of mathematics in logic, he has given primacy to mathematical intuition and forced logic to play second fiddle. This divergence of view among workers in foundations of mathematics will constitute the subject-matter of the present discussion.

The issue between Brouwer’s intuitionist school and its opponents was sharply defined with the declaration by the latter of the unlimited validity of the law of excluded middle while the former protested an ignorabimus with reference to it. The considerations which governed the stand taken were certain conceptions of the nature of mathematics and of the relation of logic to it. Now on this point a threefold rather than twofold cleavage came to pass, each camp proclaiming its own remedy for the mathematical difficulty. Two of the three schools agree on the legitimacy of the processes at issue: unrestricted definition of the questionable entities, e.g., of the set of all real numbers, and the extension of the law of excluded middle to reasoning on infinite sets. But these same schools have different answers to Brouwer’s criticisms. At least I

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THE FOUR-COLOR PROBLEM AND ITS
PHILOSOPHICAL SIGNIFICANCE *

The old four-color problem was a problem of mathematics for over a century. Mathematicians appear to have solved it to their satisfaction, but their solution raises a problem for philosophy which we might call the new four-color problem.

The old four-color problem was whether every map on the plane or sphere can be colored with no more than four colors in such a way that neighboring regions are never colored alike. This problem is so simple to state that even a child can understand it. Nevertheless, the four-color problem resisted attempts by mathematicians for more than one hundred years. From very early on it was proved that five colors suffice to color a map, but no map was ever found that required more than four colors. In fact some mathematicians thought that four colors were not sufficient and were working on methods to produce a counterexample when Kenneth Appel and Wolfgang Haken, assisted by John Koch, published a proof that four colors suffice.† Their proof has been accepted by most mathematicians.
My Plan

What logic is
Anti-exceptionalism
Quine
Practices
Rules
Definitions
WHAT LOGIC IS
Setting the Scene

PROOF THEORY

- Design and construction of different proof systems, proofs in those systems, and results about those proof systems.
- Axiomatic development of different theories. Translations between theories, reductions, embeddings …
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MODEL THEORY

- Design and construction of different classes of models.
- Model constructions for different theories. Modelling different phenomena. Independence results.
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METATHEORY

- Soundness and completeness. Limitative Results.
**Different Perspectives**

**External & Internal**

There's a difference between treating proofs and models as mathematical structures to be *analysed*, and *adopting* them.
Different Perspectives

External & Internal

There’s a difference between treating proofs and models as mathematical structures to be analysed, and adopting them.

There’s a difference between comparing different logics, and using some logic, by using a given proof to justify a conclusion, or taking a model to interpret a theory.
ANTI-
EXCEPTIONALISM
What is anti-exceptionalism?

Anti-exceptionalism about logic

Ole Thomassen Hjortland

Abstract Logic isn’t special. Its theories are continuous with science; its method continuous with scientific method. Logic isn’t a priori, nor are its truths analytic truths. Logical theories are revisable, and if they are revised, they are revised on the same grounds as scientific theories. These are the tenets of anti-exceptionalism about logic. The position is most famously defended by Quine, but has more recent advocates in Maddy (Proc Address Am Philos Assoc 76:61–90, 2002), Priest (Doubt truth to be a liar, OUP, Oxford, 2006a, The metaphysics of logic, CUP, Cambridge, 2014, Log et Anal, 2016), Russell (Philos Stud 171:161–175, 2014, J Philos Log 0:1–11, 2015), and Williamson (Modal logic as metaphysics, Oxford University Press, Oxford, 2013b, The relevance of the liar, OUP, Oxford, 2015). Although
What is anti-exceptionalism?

- Logic isn’t special.
- Logic’s theories are continuous with science.
- Logic’s methods are continuous with scientific method.
- Logic isn’t *a priori*.
- Logic’s truths are not analytic truths.
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- If logical theories are revised, they are revised on the same grounds as scientific theories.
Compare Arithmetic

- Arithmetic isn’t special.
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- Arithmetic isn’t *a priori*.
- Arithmetic’s truths are not analytic truths.
- Arithmetic theories are revisable.
- If arithmetic theories are revised, they are revised on the same grounds as scientific theories.
Here's a proof that $2 + 2 = 4$, in Robinson's Arithmetic

\[
\begin{align*}
0'' + 0'' &= (0'' + 0')' \\
0'' + 0' &= (0'' + 0)'
\end{align*}
\]

Q5

\[
\begin{align*}
(0'' + 0')' &= (0'' + 0)'' \\
0'' + 0'' &= (0'' + 0)''
\end{align*}
\]

\[\implies \]

\[
0'' + 0'' = 0'''
\]

\[
\begin{align*}
0'' + 0 &= 0'' \\
(0'' + 0) &= 0'''
\end{align*}
\]

\[\implies \]

\[
(0'' + 0)'' = 0'''
\]

\[\implies \]

\[
(0'' + 0)''' = 0''''
\]

\[\implies \]

\[
0'' + 0'' = 0''''
\]

(Q4) $x + 0 = x$  (Q5) $x + y' = (x + y)'$  ('=) $x = y / x' = y'$  (|=T) $x = y$, $y = z / x = z$
Here’s a proof that \(2 + 2 = 4\), in Robinson’s Arithmetic

\[
\begin{align*}
0'' + 0'' &= (0'' + 0')' \\
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&= 0'' + 0'' = 0'''' \\
\end{align*}
\]

\(Q4\) \(x + 0 = x\) \(Q5\) \(x + y' = (x + y)'\) \(=\) \(x = y / x' = y'\) \(=T\) \(x = y, y = z / x = z\)

Is this derivation \textit{a priori} or \textit{a posteriori}?
Here's a proof that $2 + 2 = 4$, in Robinson's Arithmetic

\[
\begin{align*}
0'' + 0'' & = (0'' + 0')' \\
\frac{0'' + 0'}{(0'' + 0')'} & = (0'' + 0)' \\
\frac{(0'' + 0)'}{(0'' + 0)'} & = 0'' + 0'' \\
0'' + 0'' & = 0''''
\end{align*}
\]

(Q4) $x + 0 = x$  \hspace{1cm} (Q5) $x + y' = (x + y)'$  \hspace{1cm} ('=) x = y / x' = y'  \hspace{1cm} (=T) x = y, y = z / x = z

Is this derivation *a priori* or *a posteriori*?

If some evidence were needed to supplement the argument, where would we add it?

Greg Restall

What's so Special about Logic?, Practices, Rules and Definitions 12 of 40
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What does this proof do?

\[
\begin{align*}
(p \rightarrow q)^3 & \quad \quad (r \rightarrow p)^2 \quad (r\rightarrow q)^1 \\
\quad \quad \quad \quad p & \quad \quad \quad \quad \quad \rightarrow E \\
\quad \quad \quad \quad q & \quad \quad \quad \quad \quad \rightarrow I^1 \\
\quad \quad \quad \quad r \rightarrow q & \quad \quad \quad \quad \quad \rightarrow I^2 \\
\quad \quad \quad \quad (r \rightarrow p) \rightarrow (r \rightarrow q) & \quad \quad \quad \quad \quad \rightarrow I^3 \\
\quad \quad (p \rightarrow q) \rightarrow ((r \rightarrow p) \rightarrow (r \rightarrow q)) &
\end{align*}
\]
What does this proof do?

\[ [r \rightarrow p]^2 \quad [r]^1 \]
\[ [p \rightarrow q]^3 \quad p \]
\[ \rightarrow E \]
\[ q \]
\[ \rightarrow I^1 \]
\[ r \rightarrow q \]
\[ \rightarrow I^2 \]
\[ (r \rightarrow p) \rightarrow (r \rightarrow q) \]
\[ \rightarrow I^3 \]
\[ (p \rightarrow q) \rightarrow ((r \rightarrow p) \rightarrow (r \rightarrow q)) \]

Is the conclusion **analytic**?

---

Greg Restall What's so Special about Logic?, Practices, Rules and Definitions 14 of 40
What does this proof do?

\[
\begin{align*}
[p \rightarrow q]^3 & \quad \Rightarrow p \quad \Rightarrow E \\
[p \rightarrow q]^3 & \quad q \quad \Rightarrow I^1 \\
[p \rightarrow q]^3 & \quad r \rightarrow q \quad \Rightarrow I^2 \\
(p \rightarrow q) & \quad ((r \rightarrow p) \rightarrow (r \rightarrow q)) \quad \Rightarrow I^3
\end{align*}
\]

Is the conclusion analytic? \(\Box\) Is the conclusion a priori?
What does this proof do?

\[
\begin{align*}
[p \to q]_3 & \quad [r]_1 \\
\frac{[p \to q]_3 \quad p}{\to E} \\
\frac{\frac{q}{\to I^1}}{r \to q} & \quad \to I^1 \\
\frac{(r \to p) \to (r \to q)}{(p \to q) \to ((r \to p) \to (r \to q))} & \quad \to I^3
\end{align*}
\]

Is the conclusion **analytic?** Is the conclusion **a priori?** Is the proof **special?**
Anti-exceptionalism about logic

Ole Thomassen Hjortland

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§13 Translating Logical Connectives

In §§7 through 11 we accounted for radical translation of occasion sentences, by approximate identification of stimulus meanings. Now there is also a decidedly different domain that lends itself directly to radical translation: that of truth functions such as negation, logical conjunction, and alternation. For this purpose the sentences put to the native for assent or dissent may be occasion sentences and standing sentences indifferently. Those that are occasion sentences will have to be accompanied by a prompting stimulation, if assent or dissent is to be elicited; the standing sentences, on the other hand, can be put without props. Now by reference to assent and dissent we can state semantic criteria for truth functions; i.e., criteria for determining whether a given native idiom is to be construed as expressing the truth function in question. The semantic criterion of negation is that it turns any short sentence to which one will assent into a sentence from which one will dissent, and vice versa. That of conjunction is that it produces compounds to which (so long as the component sentences are short) one is prepared to assent always and only when one is prepared to assent to each component. That of alternation is similar with assent changed twice to dissent.
Why, then, is a conjunction true when both conjuncts are true?
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Why is a disjunction false when both disjuncts are false?
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Why is a disjunction false when both disjuncts are false?

For the Quine of *Word and Object*, inferences like these *are a priori* valid.
Why, then, is a conjunction true when both conjuncts are true?

Why is a disjunction false when both disjuncts are false?

For the Quine of *Word and Object*, inferences like these are a priori valid.

(Not a priori in the sense that they are unrevisable, but in the sense that if the terms have the meanings we have postulated, we do not need to appeal to evidence to ground the validity of the inference.)
The constitutive and relativized a priori

... the concept of the relativized a priori, as originally formulated within the tradition of logical empiricism, was explicitly intended to prise apart two meanings that were discerned within the original Kantian conception: necessary and un revisable, true for all time, on the one hand, and “constitutive of the concept of the object of [scientific] knowledge,” on the other.

What does “and”, in this sense, mean?
Constraints

What does “and”, in this sense, mean?

What does “or”, in this sense, mean?
Constraints

What does “and”, in this sense, mean?

What does “or”, in this sense, mean?

For the Quine of *Word and Object*, it is not a bridge too far to say that principles governing these particles are *definitionally analytic*. 
Anti-anti-exceptionalism—internally

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PRACTICES
For the Quine of *Word and Object*, you locate the logical connectives by identifying their interaction with *assent* and *dissent*.
Quine’s Criteria for Negation,

The semantic criterion of negation is that it turns any short sentence to which one will assent into a sentence from which one will dissent, and vice versa.
Quine's Criteria for Negation, Conjunction

The semantic criterion of negation is that it turns any short sentence to which one will assent into a sentence from which one will dissent, and vice versa. That of conjunction is that it produces compounds to which (so long as the component sentences are short) one is prepared to assent always and only when one is prepared to assent to each component.
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These criteria are not enough to generate truth functional logic, unless supplemented.
Suppose I dissent from $p \lor \neg p$. 

Plausible condition: I never assent to and dissent from the same thing.

Conjecture: I cannot dissent from any truth-functional tautology.

Counterexample: $(p \lor \neg p) \land (q \lor \neg q)$. 

(Quine gives no conditions concerning when to dissent from a conjunction.)
Illustrating the issue

Suppose I dissent from $p \lor \neg p$.
So, I dissent from $p$ and dissent from $\neg p$.
So, I assent to $p$.
Suppose I dissent from \( p \lor \neg p \).

So, I dissent from \( p \) and dissent from \( \neg p \).

So, I assent to \( p \).

*Plausible(?) condition:* I never assent to and dissent from the same thing.
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Quine’s project in *Word and Object* involved radical translation, stimulus meaning and occasion sentences, and much besides.
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It does arrive at a radical holism, but one in which a certain amount of logic is constitutively *a priori*.
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It does arrive at a radical holism, but one in which a certain amount of logic is constitutively *a priori*.

I will not be adopting Quine’s project, but I am interested in exploring how logical concepts can be *defined* and how this explains how logic is *special*. 
Quine’s Criteria for Negation, Conjunction and Disjunction

The semantic criterion of negation is that it turns any short sentence to which one will assent into a sentence from which one will dissent, and vice versa. That of conjunction is that it produces compounds to which (so long as the component sentences are short) one is prepared to assent always and only when one is prepared to assent to each component. That of alternation is similar with assent changed twice to dissent.

A more important question: How could we tell that we have located such items in someone’s vocabulary?
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A more important question: How could we tell that we have located such items in someone’s vocabulary?

Are these criteria descriptions of behaviour, or are they better understood in terms of...
RULES
We can bind ourselves by adopting a rule

Instead of just looking for an item in our vocabulary with the desired behaviour, we could define one.
We can bind ourselves by adopting a rule

Instead of just looking for an item in our vocabulary with the desired behaviour, we could define one.

We can adopt a rule: “use ‘∧’ like this ...”
### ‘Rules’ a la Quine

<table>
<thead>
<tr>
<th>Definiendum</th>
<th>Definiens</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+\neg A$</td>
<td>$-A$</td>
</tr>
<tr>
<td>$+A \land B$</td>
<td>$+A, +B$</td>
</tr>
<tr>
<td>$-A \lor B$</td>
<td>$-A, -B$</td>
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To make sense of these, we need to say more about assertion and denial, assent and dissent.
‘Rules’ a la Quine

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<td>+¬A</td>
<td>¬A</td>
</tr>
<tr>
<td>+A ∧ B</td>
<td>+A, +B</td>
</tr>
<tr>
<td>¬A ∨ B</td>
<td>¬A, ¬B</td>
</tr>
<tr>
<td>¬A → B</td>
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</tr>
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<td>$A, B$</td>
</tr>
<tr>
<td>$-\forall x A$</td>
<td>$A[x/n]$ (n new)</td>
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<td>$+\exists x A$</td>
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</tr>
<tr>
<td>$-s = t$</td>
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To make sense of these, we need to say more about assertion and denial, assent and dissent.
Multiple Conclusions

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VERSION 1.03
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Abstract: I argue for the following four theses. (1) Denial is not to be analysed as the assertion of a negation. (2) Given the concepts of assertion and denial, we have the resources to analyse logical consequence as relating arguments with multiple premises and multiple conclusions. Gentzen’s multiple conclusion calculus can be understood in a straightforward, motivated, non-question-begging way. (3) If a broadly anti-realist or inferentialist justification of a logical system works, it works just as well for classical logic as it does for intuitionistic logic. The special case for an anti-realist justification of intuitionistic logic over and above a justification of classical logic relies on an unjustified assumption about the shape of proofs. Finally, (4) this picture of logical consequence provides a relatively neutral shared vocabulary which can help us understand and adjudicate debates between proponents of classical and non-classical logics.

* * *
Positions

\[ X : Y \]
\([X : Y]\)

\([X, A : A, Y]\) is self-defeating.
Sequents: Unfocused and Focused

\[ X \Rightarrow Y \]
Sequents: Unfocused and Focused

\[ X \vdash Y \]

\[ X \vdash A, Y \quad X, A \vdash Y \]
Structural Rules: Identity

$x, A \rightarrow A, y$
Structural Rules: Cut

\[
\frac{X \vdash A, Y \quad X, A \vdash Y}{X \vdash Y} \quad \text{Cut}
\]
DEFINITIONS
Defining Rules for Logical Concepts

\[
\begin{align*}
X, A, B & \succ Y & X \succ A, B, Y & \rightarrow Df & X \succ A, Y & \neg Df & X, A \succ B, Y & \rightarrow Df \\
X, A \land B & \succ Y & X \succ A \lor B, Y & \lor Df & X, \neg A & \succ Y & X \succ A \rightarrow B, Y & \rightarrow Df
\end{align*}
\]

\[
\begin{align*}
X \succ A(n), Y & \lor Df & X, A(n) & \succ Y & \exists Df & X, F a \succ F b, Y & = Df \\
X \succ \forall x A(x), Y & \forall Df & X, \exists x A(x) & \succ Y & X \succ a = b, Y & = Df
\end{align*}
\]

*Terms & conditions*: the singular term \( n \) (in \( \forall / \exists Df \)) and the predicate \( F \) (in \( = Df \)) do not appear below the line in those rules.

These rules can be understood as *definitions* of the concepts they introduce (below the double line).

See (Scott 1974; Došen 1980, 1989; Restall 2019).
Adopting the Rules, Applying the Definitions

\[
\text{\( \forall x (Fx \lor Gx) \rightarrow F \lor G, \exists x Gx \)}
\]

\[
\text{\( \forall x (Fx \lor Gx), \exists x Gx \rightarrow F, \exists x Gx \)}
\]

\[
\text{\( \forall x (Fx \lor Gx), G \rightarrow F, \exists x Gx \)}
\]

\[
\text{\( \forall x (Fx \lor Gx) \rightarrow F, \exists x Gx \)}
\]

\[
\text{\( \forall x (Fx \lor Gx) \rightarrow \forall x Fx, \exists x Gx \)}
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Ante-ante-exceptionalism

The rules are constitutively \textit{a priori}. 
Anti-anti-exceptionalism

The rules are constitutively *a priori*.

The derivable formulas are *definitionally analytic*.
Anti-anti-exceptionalism

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The rules are *conservatively extending* and *uniquely defining* and these are *very* special features of these logical concepts.
Anti-anti-exceptionalism

The rules are constitutively *a priori*.

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The rules are *conservatively extending* and *uniquely defining* and these are *very* special features of these logical concepts.

In *this* way, logic is quite special.
THANK YOU!

http://consequently.org/presentation/2019/
whats-so-special-about-logic-smith