What's so Special about Logic? Practices, Rules and Definitions

Greg Restall

THE UNIVERSITY OF MELBOURNE

ALICE AMBROSE LAZEROWITZ – THOMAS TYMOCZKO MEMORIAL LOGIC LECTURE

4 DECEMBER 2019 / SMITH COLLEGE

My ***Aim***

To understand *logic* better…

My ***Aim***

To understand *logic* better…

… and especially, to understand what makes logic *distinctive*.

Alice Ambrose Lazerowitz

A CONTROVERSY IN THE LOGIC OF MATHEMATICS*

THE impetus of the attack on Cantor's theory of transfinites carried assailants and defenders into an examination of the foundations of mathematics, and thence into a questioning of logic as a substructure for mathematics. Is logic basic, as Frege and Peano assumed, and are its laws final? Brouwer has answered this question with a flat no. Scorning any grounding of mathematics in logic, he has given primacy to mathematical intuition and forced logic to play second fiddle. This divergence of view among workers in foundations of mathematics will constitute the subject-matter of the present discussion.

The issue between Brouwer's intuitionist school and its opponents was sharply defined with the declaration by the latter of the unlimited validity of the law of excluded middle while the former protested an *ignorabimus* with reference to it. The considerations which governed the stand taken were certain conceptions of the nature of mathematics and of the relation of logic to it. Now on this point a threefold rather than twofold cleavage came to pass, each camp proclaiming its own remedy for the mathematical difficulty. Two of the three schools agree on the legitimacy of the processes at issue: unrestricted definition of the questionable entities, *e.g.*, of the set of all real numbers, and the extension of the law of excluded middle to reasoning on infinite sets. But these same schools have different answers to Brouwer's criticisms. At least I

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Thomas Tymoczko

THE JOURNAL OF PHILOSOPHY

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THE FOUR-COLOR PROBLEM AND ITS PHILOSOPHICAL SIGNIFICANCE *

THE old four-color problem was a problem of mathematics for over a century. Mathematicians appear to have solved it to their satisfaction, but their solution raises a problem for philosophy which we might call the *new four-color problem*.

The old four-color problem was whether every map on the plane or sphere can be colored with no more than four colors in such a way that neighboring regions are never colored alike. This problem is so simple to state that even a child can understand it. Nevertheless, the four-color problem resisted attempts by mathematicians for more than one hundred years. From very early on it was proved that five colors suffice to color a map, but no map was ever found that required more than four colors. In fact some mathematicians thought that four colors were not sufficient and were working on methods to produce a counterexample when Kenneth Appel and Wolfgang Haken, assisted by John Koch, published a proof that four colors suffice.† Their proof has been accepted by most mathemati-

My ***Plan***

What logic *is*

Anti-exceptionalism

Quine

Practices

Rules

Definitions

WHAT LOGIC *IS*

Setting the Scene

PROOF THEORY

- Design and construction of different proof systems, proofs in those systems, and results *about* those proof systems.
- Axiomatic development of different theories. Translations between theories, reductions, embeddings ...

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METATHEORY

- Soundness and completeness. Limitative Results.

External & Internal

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There's a difference between treating proofs and models as mathematical structures to be *analysed*, and *adopting* them.

There's a difference between comparing different logics, and *using* some logic, by using a given *proof* to justify a conclusion, or taking a *model* to interpret a theory.

ANTI-EXCEPTIONALISM

What is anti-exceptionalism?

Philos Stud (2017) 174:631–658
DOI 10.1007/s11098-016-0701-8

Anti-exceptionalism about logic

Ole Thomassen Hjortland[1]

Published online: 9 June 2016

Abstract Logic isn't special. Its theories are continuous with science; its method continuous with scientific method. Logic isn't a priori, nor are its truths analytic truths. Logical theories are revisable, and if they are revised, they are revised on the same grounds as scientific theories. These are the tenets of anti-exceptionalism about logic. The position is most famously defended by Quine, but has more recent advocates in Maddy (Proc Address Am Philos Assoc 76:61–90, 2002), Priest (Doubt truth to be a liar, OUP, Oxford, 2006a, The metaphysics of logic, CUP, Cambridge, 2014, Log et Anal, 2016), Russell (Philos Stud 171:161–175, 2014, J Philos Log 0:1–11, 2015), and Williamson (Modal logic as metaphysics, Oxford University Press, Oxford, 2013b, The relevance of the liar, OUP, Oxford, 2015). Although

What is anti-exceptionalism?

- Logic isn't special.
- Logic's theories are continuous with science.
- Logic's methods are continuous with scientific method.
- Logic isn't *a priori*.
- Logic's truths are not analytic truths.
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Compare Arithmetic

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- If arithmetic theories are revised, they are revised on the same grounds as scientific theories.

Here's a proof that $2 + 2 = 4$, in Robinson's Arithmetic

$$\dfrac{\dfrac{\dfrac{}{0'' + 0'' = (0'' + 0')'}\,Q5 \qquad \dfrac{\dfrac{}{0'' + 0' = (0'' + 0)'}\,Q5}{(0'' + 0')' = (0'' + 0)''}\,{}'= }{0'' + 0'' = (0'' + 0)''}\,{=}T \qquad \dfrac{\dfrac{}{0'' + 0 = 0''}\,Q4}{(0'' + 0)' = 0'''}\,{}'= }{(0'' + 0)'' = 0''''}\,{}'= }{0'' + 0'' = 0''''}\,{=}T$$

(Q4) $x + 0 = x$ (Q5) $x + y' = (x + y)'$ ($'=$) $x = y \,/\, x' = y'$ ($=T$) $x = y, y = z \,/\, x = z$

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Is this derivation *a priori* or *a posteriori*?

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Is this derivation *a priori* or *a posteriori*?

If some evidence were needed to supplement the argument, where would we add it?

External Questions and Internal Questions

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What does this proof do?

$$
\dfrac{\dfrac{\dfrac{\dfrac{[p \to q]^3 \qquad \dfrac{[r \to p]^2 \quad [r]^1}{p}\,{\to}E}{q}\,{\to}E}{r \to q}\,{\to}I^1}{(r \to p) \to (r \to q)}\,{\to}I^2}{(p \to q) \to ((r \to p) \to (r \to q))}\,{\to}I^3
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Is the conclusion *analytic*?

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Is the conclusion *analytic*? ¶ Is the conclusion *a priori*?

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Is the conclusion *analytic*? ¶ Is the conclusion *a priori*? ¶ Is the proof *special*?

QUINE

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. . . had its limits. (Word and Object)

§13 Translating Logical Connectives

In §§7 through 11 we accounted for radical translation of occasion sentences, by approximate identification of stimulus meanings. Now there is also a decidedly different domain that lends itself directly to radical translation: that of *truth functions* such as negation, logical conjunction, and alternation. For this purpose the sentences put to the native for assent or dissent may be occasion sentences and standing sentences indifferently. Those that are occasion sentences will have to be accompanied by a prompting stimulation, if assent or dissent is to be elicited; the standing sentences, on the other hand, can be put without props. Now by reference to assent and dissent we can state semantic criteria for truth functions; i.e., criteria for determining whether a given native idiom is to be construed as expressing the truth function in question. The semantic criterion of negation is that it turns any short sentence to which one will assent into a sentence from which one will dissent, and vice versa. That of conjunction is that it produces compounds to which (so long as the component sentences are short) one is prepared to assent always and only when one is prepared to assent to each component. That of alternation is similar with assent changed twice to dissent.

Constraints

Why, then, is a conjunction true when both conjuncts are true?

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Why, then, is a conjunction true when both conjuncts are true?

Why is a disjunction false when both disjuncts are false?

For the Quine of *Word and Object*, inferences like these *are a priori* valid.

(Not *a priori* in the sense that they are unrevisable,
but in the sense that *if* the terms have the meanings
we have postulated, we do not need to appeal to evidence
to ground the validity of the inference.)

The constitutive and relativized a priori

. . . the concept of the relativized a priori, as originally formulated within the tradition of logical empiricism, was explicitly intended to prise apart two meanings that were discerned within the original Kantian conception: necessary and unrevisable, true for all time, on the one hand, and "constitutive of the concept of the object of [scientific] knowledge," on the other.

— Michael Friedman, *The Dynamics of Reason* (2002)

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What does "*and*", in this sense, *mean*?

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What does "*or*", in this sense, *mean*?

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What does "*and*", in this sense, *mean*?

What does "*or*", in this sense, *mean*?

For the Quine of *Word and Object*, it is not a bridge too far
to say that principles governing these particles are *definitionally analytic*.

***Anti**-anti-exceptionalism—internally*

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PRACTICES

Assent and Dissent

For the Quine of *Word and Object*,
you locate the logical connectives
by identifying their interaction
with *assent* and *dissent*.

Quine's Criteria for Negation,

The semantic criterion of negation is that it turns any short sentence to which one will assent into a sentence from which one will dissent, and vice versa.

Quine's Criteria for Negation, Conjunction

The semantic criterion of negation is that it turns any short sentence to which one will assent into a sentence from which one will dissent, and vice versa. That of conjunction is that it produces compounds to which (so long as the component sentences are short) one is prepared to assent always and only when one is prepared to assent to each component.

Quine's Criteria for Negation, Conjunction and Disjunction

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These criteria are not enough to generate truth functional logic, unless supplemented.

Illustrating the issue

Suppose I dissent from $p \vee \neg p$.

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Suppose I dissent from $p \vee \neg p$.

So, I dissent from p and dissent from $\neg p$.

So, I assent to p.

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Counterexample: $(p \vee \neg p) \wedge (q \vee \neg q)$.

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Counterexample: $(p \vee \neg p) \wedge (q \vee \neg q)$.

(Quine gives no conditions concerning
when to *dissent* from a conjunction.)

Quine's Project

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involved radical translation, stimulus meaning
and occasion sentences, and much besides.

It does arrive at a radical holism,
but one in which a certain amount of
logic is constitutively *a priori*.

I will not be adopting Quine's project,
but I am interested in exploring how
logical concepts can be *defined*
and how this explains how logic is *special*.

Quine's Criteria for Negation, Conjunction and Disjunction

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A more important question: How could we tell that we have located such items in someone's vocabulary?

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A more important question: How could we tell that we have located such items in someone's vocabulary?

Are these criteria descriptions of behaviour, or are they better understood in terms of . . .

RULES

We can bind ourselves by adopting a rule

Instead of just looking for an item in our vocabulary with the desired behaviour, we could *define* one.

We can bind ourselves by adopting a rule

Instead of just looking for an item in our vocabulary with the desired behaviour, we could *define* one.

We can adopt a RULE: "use '$\wedge$' like *this* . . ."

'Rules' a la Quine

DEFINIENDUM	DEFINIENS
$+\neg A$	$-A$
$+A \wedge B$	$+A, +B$
$-A \vee B$	$-A, -B$

'Rules' a la Quine

DEFINIENDUM	DEFINIENS
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$+A \wedge B$	$+A, +B$
$-A \vee B$	$-A, -B$
$-A \to B$	$+A, -B$

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$-A \vee B$	$-A, -B$
$-A \rightarrow B$	$+A, -B$
$-\forall x A$	$-A[x/n]$ (n new)
$+\exists x A$	$+A[x/n]$ (n new)

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$-s = t$	$+Fs, -Ft$ (F new)

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DEFINIENDUM	DEFINIENS
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$+\exists x A$	$+A[x/n]$ (n new)
$-s = t$	$+Fs, -Ft$ (F new)

To make sense of these, we need to say more about assertion and denial, assent and dissent.

This sounds familiar

Multiple Conclusions

Greg Restall*

Philosophy Department
The University of Melbourne

restall@unimelb.edu.au

VERSION 1.03
March 19, 2004

Abstract: I argue for the following four theses. (1) Denial is not to be analysed as the assertion of a negation. (2) Given the concepts of assertion and denial, we have the resources to analyse logical consequence as relating arguments with *multiple* premises and *multiple* conclusions. Gentzen's multiple conclusion calculus can be understood in a straightforward, motivated, non-question-begging way. (3) If a broadly anti-realist or inferentialist justification of a logical system works, it works just as well for *classical* logic as it does for *intuitionistic* logic. The special case for an anti-realist justification of intuitionistic logic over and above a justification of classical logic relies on an unjustified assumption about the shape of proofs. Finally, (4) this picture of logical consequence provides a relatively neutral shared vocabulary which can help us understand and adjudicate debates between proponents of classical and non-classical logics.

* * *

Positions

$$[X : Y]$$

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$$[X : Y]$$

$[X, A : A, Y]$ is self-defeating.

Sequents: Unfocused and Focused

$$X \succ Y$$

Sequents: Unfocused and Focused

$$X \succ Y$$

$$X \succ A, Y \qquad X, A \succ Y$$

Structural Rules: Identity

$$X, A \succ A, Y$$

Structural Rules: Cut

$$\frac{X \succ A, Y \quad X, A \succ Y}{X \succ Y}\ \mathit{Cut}$$

DEFINITIONS

Defining Rules for Logical Concepts

$$\frac{X, A, B \succ Y}{X, A \wedge B \succ Y}\ \wedge Df \qquad \frac{X \succ A, B, Y}{X \succ A \vee B, Y}\ \vee Df \qquad \frac{X \succ A, Y}{X, \neg A \succ Y}\ \neg Df \qquad \frac{X, A \succ B, Y}{X \succ A \to B, Y}\ \to Df$$

$$\frac{X \succ A(n), Y}{X \succ \forall x A(x), Y}\ \forall Df \qquad \frac{X, A(n) \succ Y}{X, \exists x A(x) \succ Y}\ \exists Df \qquad \frac{X, Fa \succ Fb, Y}{X \succ a = b, Y}\ =Df$$

Terms & conditions: the singular term n (in $\forall/\exists Df$) and the predicate F (in $=Df$) do not appear below the line in those rules.

These rules can be understood as *definitions* of the concepts they introduce (below the double line).

See (Scott 1974; Došen 1980, 1989; Restall 2019).

Adopting the Rules, Applying the Definitions

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{Fn \lor Gn \succ Fn \lor Gn, \exists x Gx}{Fn \lor Gn \succ Fn, Gn, \exists x Gx}\,\lor Df
}{\forall x(Fx \lor Gx) \succ Fn, Gn, \exists x Gx}\,\forall Df
\qquad
\dfrac{\forall x(Fx \lor Gx), \exists x Gx \succ Fn, \exists x Gx}{\forall x(Fx \lor Gx), Gn \succ Fn, \exists x Gx}\,\exists Df
}{\forall x(Fx \lor Gx) \succ Fn, \exists x Gx}\,Cut
}{
\dfrac{\forall x(Fx \lor Gx) \succ \forall x Fx, \exists x Gx}{\forall x(Fx \lor Gx) \succ \forall x Fx \lor \exists x Gx}\,\lor Df
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Anti-anti-exceptionalism

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The derivable formulas are *definitionally analytic*.

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The rules are *conservatively extending* and *uniquely defining* and these are *very* special features of these logical concepts.

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The derivable formulas are *definitionally analytic*.

The rules are *conservatively extending* and *uniquely defining* and these are *very* special features of these logical concepts.

In *this* way, logic is quite special.

THANK YOU!

http://consequently.org/presentation/2019/whats-so-special-about-logic-smith